Middle-East Journal of Scientific Research 10 (2): 274-276, 2011
ISSN 1990-9233


# Application of the Homotopy Perturbation Method to the Modified BBM Equation

*Rahim Asghari*

Marine Sciences Faculty, Petroleum University of Technology, Mahmoud Abad, Iran

**Abstract:** The aim of this paper is to apply homotopy perturbation method (HPM), to obtain approximate analytic solutions of modified Bejamin-Bona-Mahoney equation. This method is a powerful device for solving a wide variety of problems The results for some values for the variables are shown in plots as well, showing the ability of the method The results reveal that the method is very effective. The restrictions of the method are mentioned.

**Key words:** Homotopy perturbation method • Modified Bejamin-Bona-Mahoney equation

## INTRODUCTION

The homotopy perturbation method (HPM) was established by Ji-Huan He in 1999 [1-8]. In this method the solution is considered as the sum of an infinite series which converges rapidly to the accurate solutions. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p\epsilon[0,1]$ which is considered as a ''small parameter''. A considerable research works have been conducted recently in applying this method to a class of linear and non-linear equations [9-21].

It can be said that He's homotopy perturbation method is a universal one, is able to solve various kinds of nonlinear functional equations. Nonlinear partial deferential equations are known to describe a wide variety of phenomena not only in physics, where applications extend over magneto. fluid dynamics, water surface gravity waves, electromagnetic radiation reactions and ion acoustic waves in plasma, but also in biology, chemistry and several other fields. The main objective of the present paper is to use the homotopy perturbation method to the Modified Bejamin-Bona-Mahoney equation [22].

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + \alpha u^2 \frac{\partial u}{\partial x} + \beta \frac{\partial^3 u}{\partial x^2 \partial t} = 0.$$

With initial conditions

$$u(x,0) = e^{-kx},$$

where $c, \alpha, \beta$ is a constant.

**Homotopy Perturbation Method:** For the purpose of applications illustration of the methodology of the proposed method, using homotopy perturbation method, we consider the following nonlinear differential equation,

$$A(u)\text{-}f(r) = 0, \qquad r \in \Omega \tag{1}$$

$$B(u, \partial u/\partial n)= 0, \qquad r \in \Gamma \tag{2}$$

where $A$ is a general differential operator, $f(r)$ is a known analytic function, $B$ is a boundary condition and $\Gamma$ is the boundary of the domain $\Omega$

The operator $A$ can be generally divided into two operators, $L$ and $N$, where $L$ is a linear, while $N$ is a nonlinear operator. Equation (1) can be, therefore, written as follows:

$$L(u) + N(u)\text{-}f(r) = 0. \tag{3}$$

Using the homotopy technique, we construct a homotopy $U(r,p):\Omega \times [0,1] \to \mathbb{R}$ which satisfies:

$$H(U,p) = (1-p)[L(U) - L(u_0))] + p[A(U) - f(r)] = 0, \quad p \in [0,1], \quad r \in \Omega, \tag{4}$$

or

$$H(U,p) = L(U) - L(u_0) + pL(u_0) + p[N(U) - f(r)] = 0. \tag{5}$$

Where $p \epsilon [0,1]$, is called homotopy parameter and $u_0$ is an initial approximation for the solution of Eq.(1), which satisfies the boundary conditions. Obviously from Esq. (4) and (5) we will have.

**Corresponding Author:** Rahim Asghari, Marine Sciences Faculty, Petroleum University of Technology, Mahmoud Abad, Iran. E-mail: meisam.mathhome@gmail.com.

$$H(U,0) = L(U)\text{-}L(u_0) = 0, \quad (6)$$

$$H(U,1) = A(U)\text{-}f(r) = 0, \quad (7)$$

we can assume that the solution of (4) or (5) can be expressed as a series in $p$, as follows:

$$U = U_0 + pU_1 + p^2U_2 + \ldots \quad (8)$$

Setting $p = 1$, results in the approximate solution of Eq. (1)

$$u = \lim_{p\to 1} U = U_0 + U_1 + U_2 + \ldots \quad (9)$$

**Homotopy perturbation method to Coupled KdV Equations:**

Example: Consider equation

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + u^2\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^2 \partial t} = 0. \quad (10)$$

With initial condition,

$$u(x,0) = e^{-x}, \quad (11)$$

To solve Eq. (10) by homotopy perturbation method, we construct the following homotopy

$$(1-p)\left(\frac{\partial U}{\partial t} - \frac{\partial u_0}{\partial t}\right) + p\left(\frac{\partial U}{\partial t} + \frac{\partial U}{\partial x} + U^2\frac{\partial U}{\partial x} + \frac{\partial^3 U}{\partial x^2 \partial t}\right) = 0,$$

or

$$\frac{\partial U}{\partial t} - \frac{\partial u_0}{\partial t} + p\left(\frac{\partial u_0}{\partial t} + \frac{\partial U}{\partial x} + U^2\frac{\partial U}{\partial x} + \frac{\partial^3 U}{\partial x^2 \partial t}\right) = 0, \quad (12)$$

Suppose the solution of Eq. (12) has the following form

$$U = U_0 + pU_1 + p^2U_2 + \ldots. \quad (13)$$

Substituting (13) into (12) and equating the coefficients of the terms with the identical powers of $p$ leads to

$$p^0: \left\{\frac{\partial U_0}{\partial t} - \frac{\partial u_0}{\partial t} = 0,\right.$$

$$p^1: \left\{\frac{\partial U_1}{\partial t} + \frac{\partial u_0}{\partial t} + \frac{\partial U_0}{\partial x} + {U_0}^2\frac{\partial U_0}{\partial x} + \frac{\partial^3 U_0}{\partial x^2 \partial t} = 0,\right.$$

$$p^2: \left\{\frac{\partial U_2}{\partial t} + + \frac{\partial U_1}{\partial x} + 2U_0U_1\frac{\partial U_0}{\partial x} + {U_0}^2\frac{\partial U_1}{\partial x} + \frac{\partial^3 U_1}{\partial x^2 \partial t} = 0,\right.$$

$$\vdots$$

$$p^j: \left\{\frac{\partial U_j}{\partial t} + \frac{\partial U_{j-1}}{\partial x} + \sum_{i=0}^{j-1}\sum_{h=0}^{j-i-1} U_iU_h\frac{\partial U_{j-h-i-1}}{\partial x} + \frac{\partial^3 U_{j-1}}{\partial x^2 \partial t} = 0,\right.$$

$$\vdots$$

We take

$$U_0 = u_0 = e^{-x}, \quad (14)$$

We have the following recurrent equations for $j = 1,2,3$

$$\left\{U_j = -\int_0^t \frac{\partial U_{j-1}}{\partial x} + \sum_{i=0}^{j-1}\sum_{h=0}^{j-i-1} U_iU_h\frac{\partial U_{j-h-i-1}}{\partial x} + \frac{\partial^3 U_{j-1}}{\partial x^2 \partial t}\,dt,\right. \quad (15)$$

With the aid of the initial approximation given by Eq. (14) and the iteration formula (15) we get the other of component as follows:

$$\begin{cases} U_1 = \dfrac{t}{e^x} + \dfrac{t}{e^{3x}} \\ U_2 = 3\dfrac{t^2}{e^{3x}} + \dfrac{5}{2}\dfrac{t^2}{e^{5x}} + \dfrac{1}{2}\dfrac{t^2}{e^x} - \dfrac{t}{e^x} - 9\dfrac{t}{e^{3x}}, \\ U_3 = \dfrac{1}{6}\dfrac{t^3}{e^x} + \dfrac{25}{2}\dfrac{t^3}{e^{5x}} + \dfrac{49}{6}\dfrac{t^3}{e^{7x}} + \dfrac{9}{2}\dfrac{t^3}{e^{3x}} - 85\dfrac{t^2}{e^{5x}} - 42\dfrac{t^2}{e^{3x}} - \dfrac{t^2}{e^x} + 81\dfrac{t}{e^{3x}} + \dfrac{t}{e^x}, \\ \vdots \end{cases}$$

Approximate solution of (10) can be obtained by setting $p = 1$

$$u = \lim_{p\to 1} U = U_0 + U_1 + U_2 + \ldots.$$

Suppose $\left\{u^* = \sum_{j=0}^{3} U_j,\right.$ the results are presented in Fig 1.

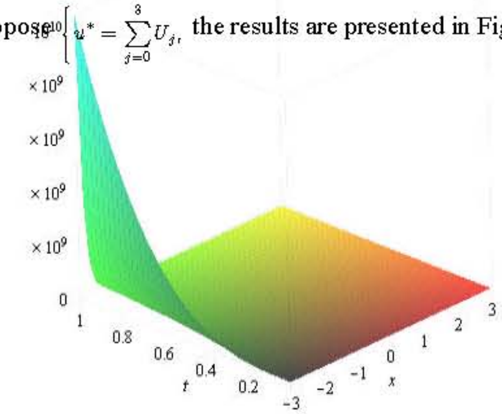


Fig. 1: The numerical results for $u^*(x,t)$, are, respectively

## CONCLUSIONS

In this article, He's homotopy perturbation method has been successfully applied to find the solution of nonlinear modified Bejamin – Bona-Mahoney equation. The analytical approximation to the solution are reliable and confirms the power and ability of the He's homotopy perturbation method as an easy device for computing the solution of a non-linear equations. The computations associated with the examples in this Letter were performed using Maple 9.

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