Middle-East Journal of Scientific Research 10 (2): 270-273, 2011
ISSN 1990-9233

Analytic Solution for Newell-Whitehead-Segel Equation by Differential Transform Method

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Abstract: In this article differential transform method (DTM) is considered to solve Newell-Whitehead-Segel equation. Differential transform method (DTM) can easily be applied to many linear and nonlinear problems and is capable of reducing the size of computational work. The fact that suggested technique solves problems without using Adomian's polynomials is a clear advantage of this algorithm over the decomposition method.

Key words: Differential transform method · Newell-Whitehead-Segel equation

INTRODUCTION

The basic idea of differential transform method (DTM) was initially introduced by Zhou [1] in 1986. Its main application therein was to solve both linear and nonlinear initial value problems arising in electrical circuit analysis. DTM is a technique that uses Taylor series for the solution of differential equations in the form of a polynomial. It is different from the high-order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally expensive especially for high order equation. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. In recent years researchers have applied the method to various linear and nonlinear problems for example it was applied to partial differential equations [2], to integro-differential equations [3], to two point boundary value problems [4], to differential-algebraic equations [5], to the KdV and mKdV equations [6], to the Schrödinger equations [7] and to fractional differential equations [8]. In recent years, special equations of composite type have received attention in many papers. In this paper, we consider homogeneous Newell-Whitehead-Segel equation [10] of the following type

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + au - bu^q \tag{1}$$

Where k, b and α are real numbers with $k > 0$ and q is a positive integer. In Eq. (1) the first term on the left hand side, $\frac{\partial u}{\partial t}$, expresses the variations of $u(x,t)$ with time at a fixed location, the first term on the right hand side, $\frac{\partial^2 u}{\partial x^2}$, expresses the variations of with spatial variable x at a specific time and the remaining terms on the right hand side, $\alpha u - bu^q$, takes into account the effect of the source term.

Basic Idea of Differential Transform Method: The basic definitions and fundamental operations of the two-dimensional differential transform are defined in as follows [2-3]. The differential transform function of the function $u(x,y)$ is the following form.

$$U(k,h) = \frac{1}{k!h!}\left[\frac{\partial^{k+h}u(x,y)}{\partial x^k \partial y^h}\right]_{(x_0,y_0)}, \tag{2}$$

Where (x,y) is the original function and $U(k,h)$ is the transformed function.
The inverse differential transform of $U(k,h)$ is defined as

$$u(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U(k,h)(x-x_0)^k (y-y_0)^h. \tag{3}$$

When (x_0,y_0) are taken as (0,0) the function $u(x,y)$, (3), is expressed as the following

$$u(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \frac{1}{k!h!}\left[\frac{\partial^{k+h}u(x,y)}{\partial x^k \partial y^h}\right]_{(0,0)} x^k y^h. \tag{4}$$

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Table 1:

Original function	Transformed function
$u(x,y) = g(x,y) \pm h(x,y)$	$U(k,h) = G(k,h) \pm H(k,h)$
$u(x,y) = \lambda g(x,u)$	$U(k,h) = \lambda G(k,h)$
$u(x,y) = \partial g(x,y) / \partial x$	$U(k,h) = (k+1)G(k+1,h)$
$u(x,y) = \partial g(x,y) / \partial y$	$U(k,h) = (h+1)G(k,h+1)$
$u(x,y) = x^m y^n$	$U(k,h) = \delta(k-m, h-n) = \begin{cases} 1, & k=m, h=n \\ o & otherwise \end{cases}$
$u(x,y) = \partial^{r+s} g(x,y) / \partial x^r \partial y^s$	$U(k,h) = (k+1)...(k+r)(h+1)...(h+s)G(k+r, h+s)$
$u(x,y) = g(x,y)h(x,y)$	$U(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{h} G(r,h-s)H(k-r,s)$

In real applications when the general term of the series cannot be recognized, the following truncated series can be considered. Eq. (4) implies that the concept of two-dimensional differential transform is derived from two-dimensional Taylor series expansion. In this study we use the lower case letters to represent the original functions and upper case letters to stand for the transformed functions (T-functions). From the definitions of Eqs. (2) and (3), it is readily proved that the transformed functions comply with the following basic mathematical operations.

The fundamental mathematical operations performed by two-dimensional differential transform method can readily be obtained and are listed in Table 1.

Numerical Example: In this section, Differential transform method (DTM) will be applied for solving Newell-Whitehead-Segel equation. The results reveal that the method is very effective and simple.

Example 1: Consider the linear equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 2u \quad (5)$$

Subject to the initial conditions

$$u(x,00 = e^x \quad (6)$$

Taking the differential transform of (5), then

$$(h+1)U(k, h+1) = (k+1)(k+2)U(k+2, h) - 2U(k,h) \quad (7)$$

From the initial condition given by Eq. (6), we obtained.

$$U(k,0) = \frac{1}{k!} \quad (8)$$

Substituting Eqs. (8) in Eq. (7), all spectra can be found as.

$$U(k,h) = \frac{(-1)^h}{k!h!}.$$

Substituting all $U(k,h)$ into Eq. (4), the series following solution form can be obtained.

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \frac{(-1)^h}{k!h!} x^k t^h = (1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots)(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \ldots),$$

this series has the closed form e^{x-t} which is the exact solution of the problem.

Example 2: Consider the following Newell-Whitehead-Segel equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2u - 3u^2 \quad (9)$$

Subject to a constant initial condition

$$u(x,0) = \lambda \quad (10)$$

Taking the differential transform of (9), then

$$(h+1)U(k,h+1) = (k+1)(k+2)U(k+2,h) + 2U(k,h) - 3\sum_{r=0}^{k}\sum_{s=0}^{h} U(r,h-s)U(k-r,s). \quad (11)$$

From the initial condition (10) we can write

$$\begin{cases} U(0,0) = \lambda, \\ U(k,0) = 0. \quad k = 1,2,\ldots \end{cases} \quad (12)$$

Substituting Eqs. (12) in Eq. (11), all spectra can be found as.

$$U(k,h) = 0. \quad for\ k = 1,2,\ldots$$
$$U(0,1) = \lambda(2-3\lambda),$$
$$U(0,2) = \frac{1}{2!}2\lambda(2-3\lambda)(1-3\lambda),$$
$$U(0,3) = \frac{1}{3!}2\lambda(2-3\lambda)(27\lambda^2-18\lambda+2),$$
$$\vdots$$

Substituting all $U(k,h)$ into Eq. (4), the following series solution will be obtained

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U(k,h)x^k t^h =$$
$$\lambda + \lambda(2-3\lambda)t + 2\lambda(2-3\lambda)(1-3\lambda)\frac{t^2}{2!} + 2\lambda(2-3\lambda)$$
$$(27\lambda^2-18\lambda+2)\frac{t^2}{3!} + \ldots = \frac{-\frac{2}{3}\lambda e^{2t}}{-\frac{2}{3}+\lambda-\lambda e^{2t}},$$

Which is an exact solution.

Example 3: Consider the two-dimensional homogeneous equation with variable coefficients.

$$e^x\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u + e^{-x}u^2, \tag{13}$$

Subject to the initial condition

$$u(x,0) = -e^x, \tag{14}$$

Taking the differential transform of (13), then

$$\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)\frac{1}{r!}\delta(h-s)U(k-r+1,s) =$$
$$(k+1)(k+2)U(k+2,h) - U(k,h) +$$
$$\sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}\frac{(-1)^k}{k!}\delta(h-s-p)U(t,s)U(k-r-t,p). \tag{15}$$

From the initial condition given by Eq. (14), we obtained

$$U(k,0) = -\frac{1}{k!}, \tag{16}$$

Substituting Eqs. (16) in Eq. (15), all spectra can be found as.

$$U(k,h) = \begin{cases} \frac{-1}{k!} & for\ h = 0, \\ \frac{1}{k!} & for\ h = 1, \\ 0 & otherwise. \end{cases}$$

Substituting all $U(k,h)$ into Eq. (4), the series following solution form can be obtained.

$$u(x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty}U(k,h)x^k t^h = e^x(t-1),$$

Which is an exact solution of the problem.

CONCLUSION

In this paper, the differential transform method (DTM) has been successfully employed to obtain analytic solution for various types of Newell-Whitehead-Segel equation. The method is effective, easy to use and reliable and the main benefit of the method is to offer the analytical approximation and, in many cases an exact solution, in a rapid convergent series form. Computations in this paper are performed using Maple 13.

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