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Chaos Synchronization of Identical Novel Chaotic Systems via Adaptive Control

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Abstract: This paper presents a new approach to solve synchronization problem of a large class of chaotic systems. The synchronization problem for this class of nonlinear systems is revisited from a control perspective and it is argued that the problem can be viewed as an adaptive control problem. In this regards, a new adaptive control method is proposed and then it is applied to the three novel chaotic systems including a 4-D (hyperchaotic) system and the two 3-D systems. The 4-D system represents a new four-wing hyper-chaotic attractor whereas the two 3-D systems show the transverse butterfly attractor and also they can be realized with an electronic oscillator circuit. Based on the Lyapunov stability theorem and Barbalat's lemma, it is shown that the error of the synchronization asymptotically converges to zero as well as time goes to infinity. Simulation results show effectiveness of our propositions.

Key words: Chaos · Adaptive synchronization · Lyapunov stability · Identical systems

INTRODUCTION

In recent years, due to potential application of chaos synchronization in various fields, it has received much attention from researchers. Some practical applications of the chaos synchronization are secure communications, biological science, chemical processes and optics. The first chaos synchronization scheme was introduced by Pecora and Carroll in [1]. Then, many researchers became interested to study on this issue such that a lot of investigations were represented in literature which many of them are based on the control techniques. These include adaptive control [2], backstepping method [3], sliding mode control [4], observer based control method [5] and so on. Among them, since in practical situations, knowledge of exact parameters of the model may not exist, the adaptive control methods have been huge welcomed by the authors such that there are a lot of works concerned with it in literature. For example C. Zhu in [6] has performed the adaptive synchronization of two novel hyperchaotic systems with partially unknown parameters. Also, the synchronization action between hyperchaotic Chen system and generalized Henon–Heiles system was investigated by X. Wu *et al.* in [7] wherein only parameters of the drive system are assumed to be unknown. Y. Wu *et al.* [8] have presented an adaptive method to synchronize special identical chaotic systems that are called *T* system. Another work was represented by X. Chen and J. Lu [9]. In that paper, the synchronization action between Lorenz-Chen, Chen-Lu and Lu-Rossler systems are investigated in three examples. Also, the synchronization of the identical novel hyperchaotic systems reported in [10] was investigated by X. Zhou *et al.* [11] via adaptive control method. However, in the above mentioned works we see that the main results of the papers were devoted to the special chaotic systems and they have not introduced general method for other systems. Nevertheless, in some papers, for general cases, several adaptive synchronization methods are proposed. For example, H. Zhang *et al.* [12] introduced a theorem which gives control and updating laws to synchronize non-identical fully unknown chaotic systems. However, theorem in [12] does not cover identical systems such as what considered in [13]. In the same way, in [14] a new adaptive method has been proposed that is more general than the theorem in [12]. W. Xu *et al.* in [15] have also introduced two adaptive chaos synchronization methods for a class of chaotic systems. First method considers the identical systems and second is concerned with the case of non-identical different order chaotic systems. In this class one can find several papers such as [16].

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This article presents a new adaptive control method to synchronize identical chaotic systems with fully unknown parameters. In our proposed method, only unknown parameters of the slave system are adapted and they are simultaneously used both in the input control and in that system. Moreover, this approach is applied to some novel chaotic systems expressed in [17-19]. To the best of our knowledge, no work is reported related to the adaptive synchronization of these systems until now. In addition, computer simulation results are presented to verify the effectiveness of the proposed method.

This paper is organized as follows. In Section 2, after statement of the problem, the new adaptive synchronization method for two identical chaotic systems has been introduced in a theorem. In Section 3, the three novel chaotic systems will be expressed and the proposed method will be applied to them. Also the simulation results are represented there. Finally, Section 4 represents some conclusions.

System Mathematical Model and Problem Description: In this section synchronization of two identical chaotic systems in a general form has been investigated via adaptive control method. To this aim, let us consider the drive system dynamic is given in the form as follows

$$\dot{x} = f(x) + F(x)\alpha \tag{1}$$

Where $x \in R^n$ is the drive state vector and $\alpha \in R^m$ is the unknown parameter vector of the drive system. $f(x)$ is an $n\times 1$ vector valued function and $F(x)$ is an $n\times m$ matrix valued function. On the other hand, the response system dynamic is

$$\dot{y} = f(y) + F(y)\tilde{\alpha} + u(t) \tag{2}$$

Where $x \in R^n$ is the slave state vector, $\tilde{\alpha} \in R^m$ is the uncertain parameter of the response system that is to be estimated and $u(t) \in R^n$ is the control input vector.

Remark 1: Note that the dynamic equation of many chaotic systems can be described by (1) and/or (2). For example Chen, Lorenz, Lu, Rossler systems are the important samples of them.

Here, our objective is to design controller, $u(t)$, such that the response system state, $y(t)$, follows the drive system state, $x(t)$. In other words, by defining $e(t) = y(t) - x(t)$, to synchronize systems (1) and (2), we should have

$$\lim_{t\to\infty} \|e(t)\| \to 0 \tag{3}$$

Where ||.|| denotes a norm operator. According to (1) and (2), the error dynamic can be written as follows

$$\begin{aligned}\dot{e}(t) &= \dot{y}(t) - \dot{x}(t) \\ &= f(y(t)) - f(x(t)) + [F(y(t))]\tilde{\alpha} - [F(x(t))]\alpha + u(t)\end{aligned} \tag{4}$$

So, it is rather easily shown that the optimal control to perform the synchronization action can be designed as

$$u(t) = f(x(t)) - f(y(t)) + [F(x(t))]\alpha - [F(y(t))]\tilde{\alpha} - Ke \tag{5}$$

Where K is $n\times n$ positive definite matrix. However, the controller in (5) is not an applicable input control because it contains unknown parameter α and $\tilde{\alpha}$. In order to remove this deficiency, a suitable approach is to use the estimation of the unknown parameters. For this reason, in the following theorem, we have proposed a new adaptive controller in which only the unknown parameters of the response system, $\tilde{\alpha}$, are estimated.

Theorem: The two identical chaotic systems (1) and (2) can be synchronized globally and asymptotically if the controller $u(t)$ is selected as

$$u(t) = f(x(t)) - f(y(t)) + \left[F(x(t)) - F(y(t))\right]\tilde{\alpha} - Ke(t) \tag{6}$$

such that $\tilde{\alpha}$ is adapted by

$$\dot{\tilde{\alpha}} = -[F(x(t))]^T e(t) \tag{7}$$

Where K is an $n\times n$ positive definite constant matrix.

Remark: Notice that the estimation parameter, $\tilde{\alpha}$, is simultaneously used both in the response system and in the controller.

Proof: By replacing proposed adaptive controller (6) in error dynamic (4), we have

$$\dot{e}(t) = [F(x(t))](\tilde{\alpha} - \alpha) - Ke(t) \tag{8}$$

Now, to show the asymptotic stability of (8), let us consider the following Lyapunov candidate function

$$V(t) = \frac{1}{2}(e^2 + \hat{\alpha}^2) \quad (9)$$

Where $\hat{\alpha} = \tilde{\alpha} - \alpha$. By taking derivation from V with respect to the time and making use of (8) and updating law (7) it can be shown that

$$\begin{aligned}\dot{V}(t) &= \dot{e}^T(t)e(t) + \hat{\alpha}^T\dot{\tilde{\alpha}} \\ &= \hat{\alpha}^T[F(x)]^T e(t) - \hat{\alpha}^T[F(x)]^T e(t) - e^T(t)K\,e(t) \\ &= -e^T K e \le 0 \end{aligned} \quad (10)$$

Since the Lyapunov function (9) is a positive definite function and its derivative is negative semi-definite, we can conclud that $e(t), \hat{\alpha} \in L_\infty$ where L_∞ denotes a normed linear space consisting any matrix-valued function with bounded infinity-norm. On the other hand, for the positive definite matrix K we can assume that $\gamma I \le K$, where γ is a positive constant and therefore

$$-\gamma e^T(t)e(t) \le -e^T(t)Ke(t) \quad (11)$$

By integrating from above equation we have

$$-\gamma\int_0^t e^T(\tau)e(\tau)d\tau \le \int_0^t \dot{V}(\tau)d\tau \quad (12)$$

and so

$$\int_0^t e^T e dt \le \frac{1}{\gamma}\left[V(0) - V(t)\right] \le \frac{V(0)}{\gamma} \quad (13)$$

Thus, it is obvious that $e \in L_2$ (L_2 is a normed linear space with bounded 2-norm). From (4) we have $\dot{e} \in L_\infty$. According to Barbalat's lemma [20], we can conclude that $\lim_{t\to\infty} e(t) = 0$ which means the two identical chaotic systems (1) and (2) are synchronized globally and asymptotically. This completes the proof.

Application to Three Novel Chaotic Systems: In this section, to show the effectiveness of our proposed adaptive method, we want to apply it to the three new chaotic systems. The first system is reported by S. Cang *et al.* in [17] which is a 4-D hyperchaotic system with four wing attractors. Two other novel chaotic systems recently introduced in [18] and [19]. However, to the best of our knowledge, any adaptive synchronization for these systems has been presented in literature up to now. In the sequel of the paper, these chaotic systems are briefly reviewed and the simulation results are represented.

First Example: The first novel chaotic system that we want to consider has been reported in [17]. The dynamic equation of the system is shown below.

$$\begin{cases}\dot{x} = -ax - ew + yz \\ \dot{y} = by + xz \\ \dot{z} = cz + fw - xy \\ \dot{w} = dw - gz\end{cases} \quad (14)$$

Where x, y, z and w are the state variables and a, b, c, d, e, f and g are the constant numbers. S. Cang *et al.* in [17] showed that the nonlinear system (14) can generate four-wing hyper chaotic attractors and also has five equilibrium points where one of them is the origin and other equilibrium points are symmetrically situated with respect to y-axis. In Fig. 1, the state trajectory of the system are illustrate in 2-D view when it originates from two different initial conditions (1,1,1,1) (shown as solid line) and (1,–1,1,1) (shown as dashed line). The constant parameters are selected as $a = 50$, $b = -5.5$, $c = 10$, $d = 0.2$, $e = 10$, $f = 16$ and g = 0.5.

Numerical Examples for The 4-D System: Consider the drive and response system dynamics be given respectively as follows

$$\begin{cases}\dot{x}_1 = -ax_1 - ew_1 + y_1z_1 \\ \dot{y}_1 = by_1 + x_1z_1 \\ \dot{z}_1 = cz_1 + fw_1 - x_1y_1 \\ \dot{w}_1 = dw_1 - gz_1\end{cases} \quad (15)$$

and

$$\begin{cases}\dot{x}_2 = -\tilde{a}x_2 - \tilde{e}w_2 + y_2z_2 + u_1 \\ \dot{y}_2 = \tilde{b}y_2 + x_2z_2 + u_2 \\ \dot{z}_2 = \tilde{c}z_2 + \tilde{f}w_2 - x_2y_2 + u_3 \\ \dot{w}_2 = \tilde{d}w_2 - \tilde{g}z_2 + u_4\end{cases} \quad (16)$$

Where $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{e}, \tilde{f}$ and $\tilde{g}$ are the estimations of the unknown parameters a, b, c, d, e, f and g respectively. Also, u_1, u_2, u_3 and u_4 are the input control variables. Note that, according to (1) and (2) we can define.

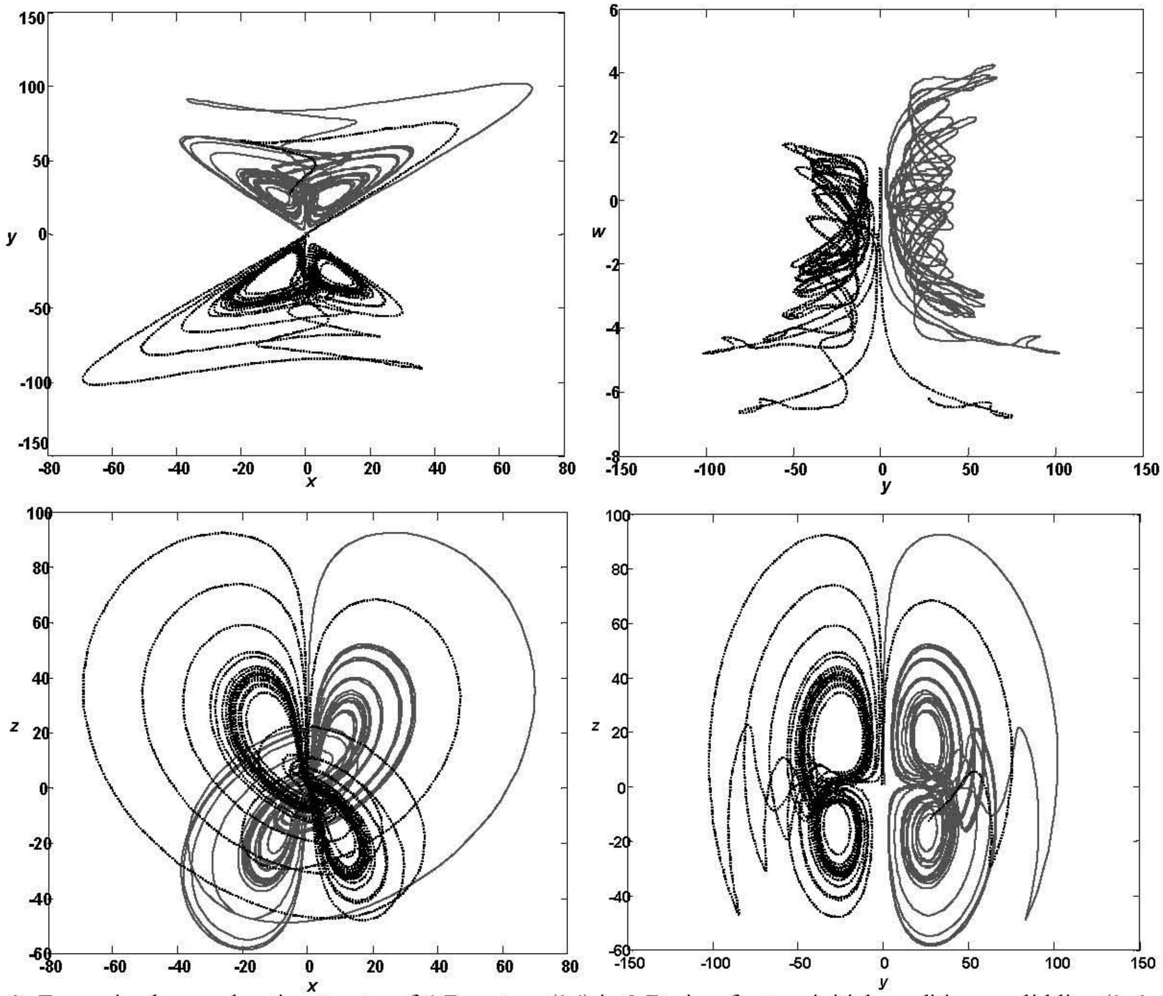

Fig. 1: Four-wing hyper-chaotic attractor of 4-D system (14) in 2-D view for two initial conditions, solid line (1, 1, 1, 1) and dashed line (1, -1, 1, 1).

$$x=\begin{bmatrix}x_1\\y_1\\z_1\\w_1\end{bmatrix},\ f(x)=\begin{bmatrix}y_1z_1\\x_1z_1\\-x_1y_1\\0\end{bmatrix},\ F(x)=\begin{bmatrix}-x_1&0&0&0&-w_1&0&0\\0&y_1&0&0&0&0&0\\0&0&z_1&0&0&w_1&0\\0&0&0&w_1&0&0&-z_1\end{bmatrix},\ \alpha=\begin{bmatrix}a\\b\\c\\d\\e\\f\\g\end{bmatrix} \tag{17}$$

and

$$y=\begin{bmatrix}x_2\\y_2\\z_2\\w_2\end{bmatrix},\ f(y)=\begin{bmatrix}y_2z_2\\x_2z_2\\-x_2y_2\\0\end{bmatrix},\ F(y)=\begin{bmatrix}-x_2&0&0&0&-w_2&0&0\\0&y_2&0&0&0&0&0\\0&0&z_2&0&0&w_2&0\\0&0&0&w_2&0&0&-z_2\end{bmatrix},\ \tilde{\alpha}=\begin{bmatrix}\tilde{a}\\\tilde{b}\\\tilde{c}\\\tilde{d}\\\tilde{e}\\\tilde{f}\\\tilde{g}\end{bmatrix},$$

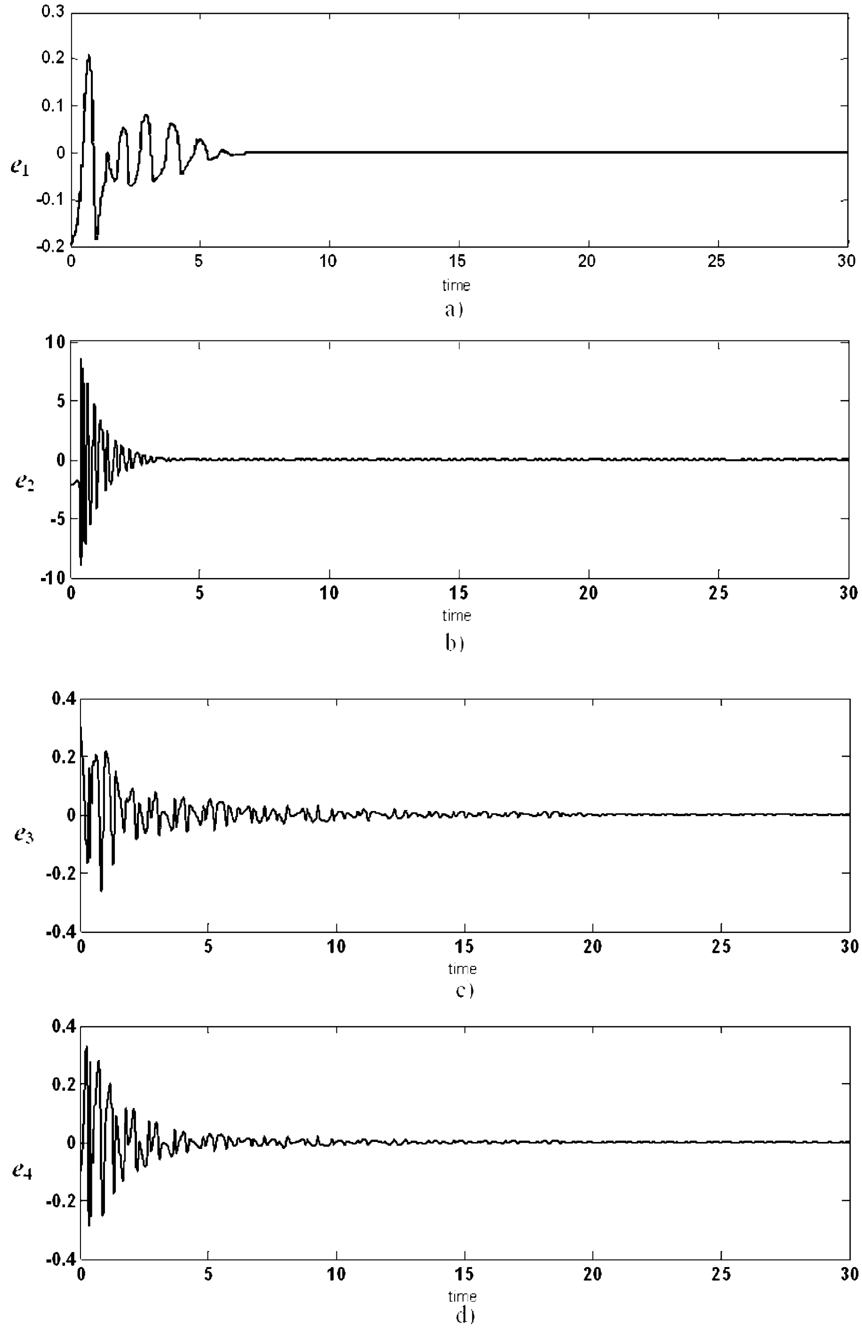

Fig. 2: Errors of the synchronization between 4-D systems, a) $e1$, b) e_2, c) e_3 and d) e_4

$$u(t)=\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{18}$$

Then, noting to the theorem and (6), in order to synchronize (16) with (15), the control action, $u(t)$, is

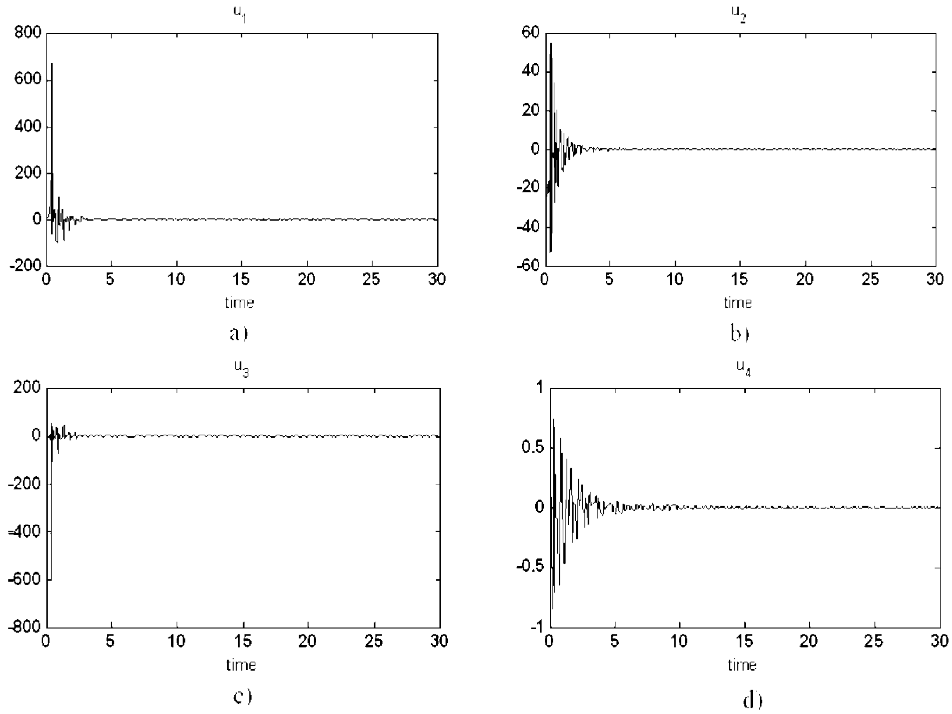

Fig. 3: Input controls in the first example, a) u_1, b) u_2, c) u_3 and d) u_4

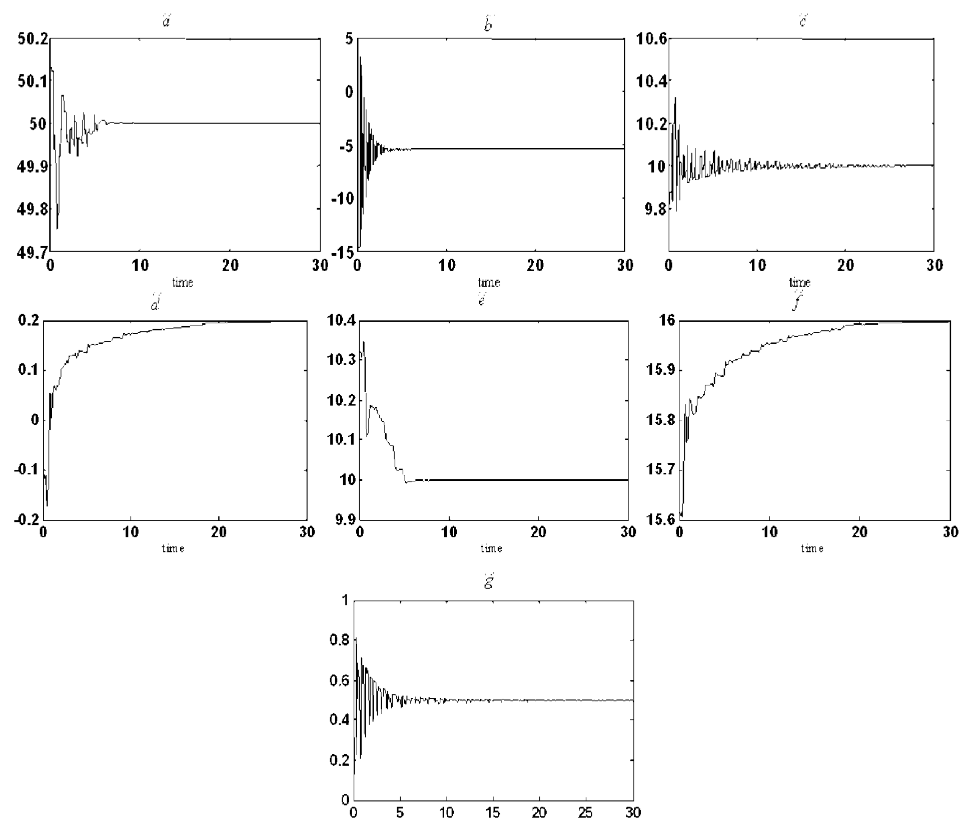

Fig. 4: Estimation of unknown parameters for the 4-D system

$$\begin{cases} u_1 = y_1 z_1 - y_2 z_2 + \tilde{a}(x_2 - x_1) + \tilde{e}(w_2 - w_1) - e_1 \\ u_2 = x_1 z_1 - x_2 z_2 + \tilde{b}(y_1 - y_2) - e_2 \\ u_3 = -x_1 y_1 + x_2 y_2 + \tilde{c}(z_1 - z_2) + \tilde{f}(w_1 - w_2) - e_3 \\ u_4 = \tilde{d}(w_1 - w_2) + \tilde{g}(z_2 - z_1) - e_4 \end{cases} \tag{19}$$

Where $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$ and $e_4 = w_2 - w_1$ and also, K = diag[2.5 2.5 2.5 2.5]. In addition, according to (7), the updating laws are

$$\begin{cases} \dot{\tilde{a}} = x_1 e_1 \\ \dot{\tilde{b}} = -y_1 e_2 \\ \dot{\tilde{c}} = -z_1 e_3 \\ \dot{\tilde{d}} = -w_1 e_4 \\ \dot{\tilde{e}} = w_1 e_1 \\ \dot{\tilde{f}} = -w_1 e_3 \\ \dot{\tilde{g}} = z_1 e_4 \end{cases} \tag{20}$$

In our simulation, the initial condition of the drive system is (1, 1, 1, 1) while the response system starts with (0.8, -0.9, 1.3, 0.9). Moreover, the constant parameters of the drive system are a = 50 b = –5.5, c = 10, d = 0.2, e = 10, f = 16 and g = 0.5. You can see the results in Fig. 2 to 4.

Fig. 2. illustrates the errors of the synchronization with respect to time. The input controls and the parameter estimations are also illustrated in Fig. 3. and 4. Moreover, the initial conditions for the adaptation laws are chosen as $\tilde{a}(0) = 50.1$, $\tilde{b}(0) = -5$ $\tilde{c}(0) = 9.96$, $\tilde{d}(0) = -0.114$, $\tilde{e}(0) = 10.34$, $\tilde{f}(0) = 15.64$ and $\tilde{g}(0) = 0.14$.

The simulations in this paper is run using runge-kutta method with step size 0.001.

Second Example: A new 3-D autonomous chaos system that was reported in [18] is described as follows

$$\begin{cases} \dot{x} = -ax - ey^2 \\ \dot{y} = by - kxz \\ \dot{z} = -cz + mxy \end{cases} \tag{21}$$

Where x, y, z are the state variables and a, b, c, e, k and m are the constant parameters. This chaotic system has a strong chaotic attractor when a = 1 b = 2.5, c = 5, e = 1, k = 4 and m = 4 similar to the Lorenz system but not equivalent such that this is a transverse butterfly-shaped attractor [18] (Fig. 5). Doing some computations, it can be shown that chaotic system (21) has five equilibrium points which are

$$(0,0,0),\ (\pm 1.25, \pm 1.118, -0.559), (\pm 1.25, \mp 1.118, 0.559) \tag{22}$$

Moreover, it is shown in [18] that all equilibrium points are unstable saddle focus-nodes.

Numerical Example for the First 3-D System: In this example, the drive and response system dynamics be respectively given as

$$\begin{cases} \dot{x}_1 = -a x_1 - e y_1^2 \\ \dot{y}_1 = b y_1 - k x_1 z_1 \\ \dot{z}_1 = -c z_1 + m x_1 y_1 \end{cases} \tag{23}$$

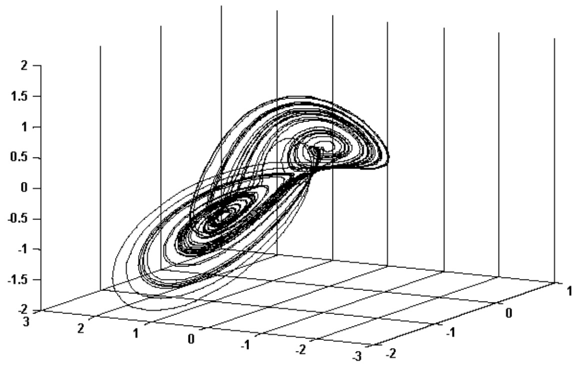

Fig. 5: Transverse butterfly-attractive of system (14) in 3-D view

$$\begin{cases} \dot{x}_2 = -\tilde{a}x_2 - \tilde{e}y_2^2 + u_1(t) \\ \dot{y}_2 = \tilde{b}y_2 - \tilde{k}x_2z_2 + u_2(t) \\ \dot{z}_2 = -\tilde{c}z_2 + \tilde{m}x_2y_2 + u_3(t) \end{cases} \tag{24}$$

Where $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{e}, \tilde{k}$ and $\tilde{m}$ are estimations of *a*, *b*, *c*, *e*, *k* and *m* respectively. In this example it is assumed that the unknown parameters of the drive system are $a = 1$, $b = 2.5$, $c = 5$, $e = 1$, $k = 4$ and $m = 4$. Also, the initial condition of the drive system is (0.2, 0, 0.5) while the response system starts with (0.15, 0.02, 0.49). With respect to (1) and (2) we can define

$$f(x(t)) = 0,\ F(x(t)) = \begin{bmatrix} -x_1 & 0 & 0 & -y_1^2 & 0 & 0 \\ 0 & y_1 & 0 & 0 & -x_1z_1 & 0 \\ 0 & 0 & -z_1 & 0 & 0 & x_1y_1 \end{bmatrix},\ \alpha = \begin{bmatrix} a & b & c & e & k & m \end{bmatrix}^T$$

and

$$f(y(t)) = 0,\ F(y(t)) = \begin{bmatrix} -x_2 & 0 & 0 & -y_2^2 & 0 & 0 \\ 0 & y_2 & 0 & 0 & -x_2z_2 & 0 \\ 0 & 0 & -z_2 & 0 & 0 & x_2y_2 \end{bmatrix},\ \tilde{\alpha} = \begin{bmatrix} \tilde{a} & \tilde{b} & \tilde{c} & \tilde{e} & \tilde{k} & \tilde{m} \end{bmatrix}^T$$

Where $x = [x_1\ y_1\ z_1]^T$ and $y = [x_2\ y_2\ z_2]^T$. So, with respect to the theorem and from (6) and (7) and if we select

$$K = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{25}$$

then the adaptive controls and the updating laws are designed respectively as follows

$$\begin{cases} u_1(t) = (\tilde{a} - 1)e_1 + (-y_1^2 + y_2^2)\tilde{e} \\ u_2(t) = -(\tilde{b} + 1)e_2 + (-x_1z_1 + x_2z_2)\tilde{k} \\ u_3(t) = (\tilde{c} - 1)e_3 + (x_1y_1 - x_2y_2)\tilde{m} \end{cases} \tag{26}$$

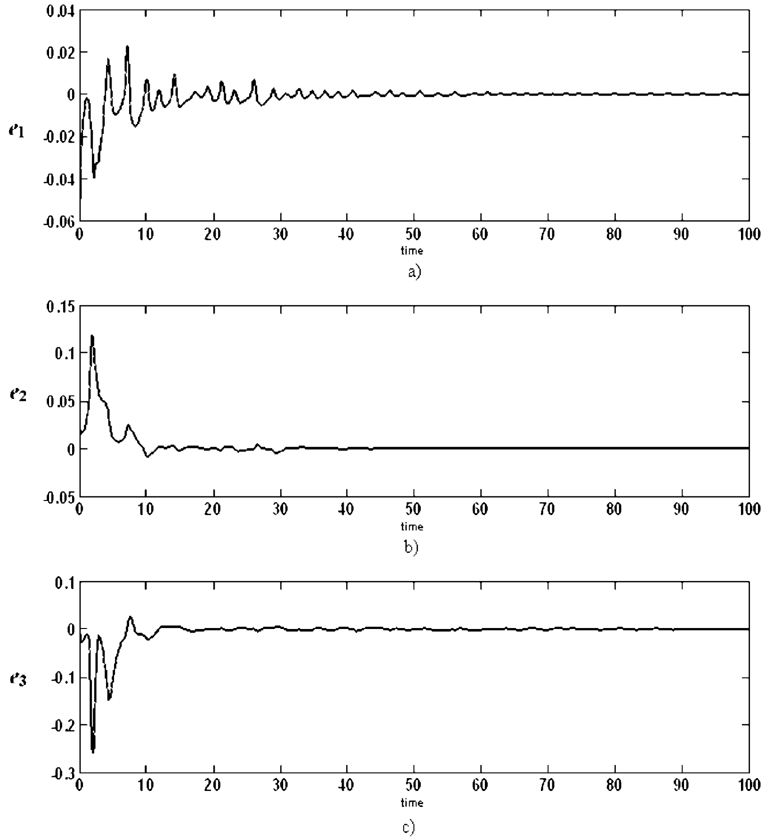

Fig. 6: Errors of the synchronization in the first example, a) e_1, b) e_2 and c) e_3

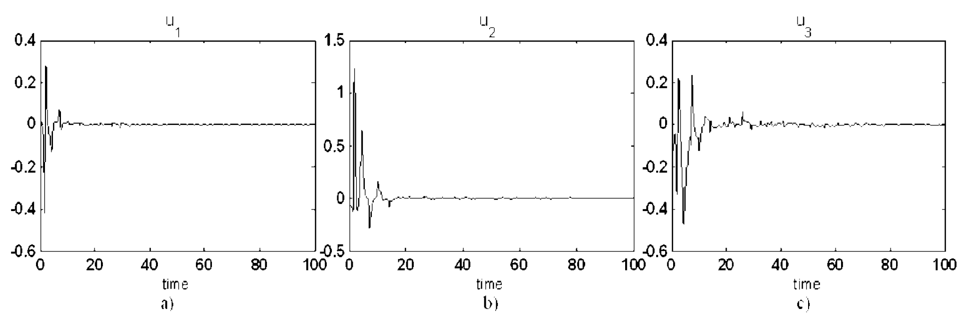

Fig. 7: Input controls in the first example, a) u_1, b) u_2 and c) u_3

$$\begin{cases} \dot{\tilde{a}} = x_1 e_1 \\ \dot{\tilde{b}} = -y_1 e_2 \\ \dot{\tilde{c}} = z_1 e_3 \\ \dot{\tilde{e}} = y_1^2 e_1 \\ \dot{\tilde{k}} = x_1 z_1 e_2 \\ \dot{\tilde{m}} = -x_1 y_1 e_3 \end{cases} \tag{27}$$

Where $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$ and $e_3 = z_2 - z_1$. In this simulation, the initial conditions of the adaptation parameters are chosen as $\tilde{a}(0) = 0.9$, $\tilde{b}(0) = 2.4$ $\tilde{c}(0) = 5.3$, $\tilde{e}(0) = 1$, $\tilde{k}(0) = 4.1$ and $\tilde{m}(0) = 3.8$. The simulation results are shown in Fig. 6. to 8.

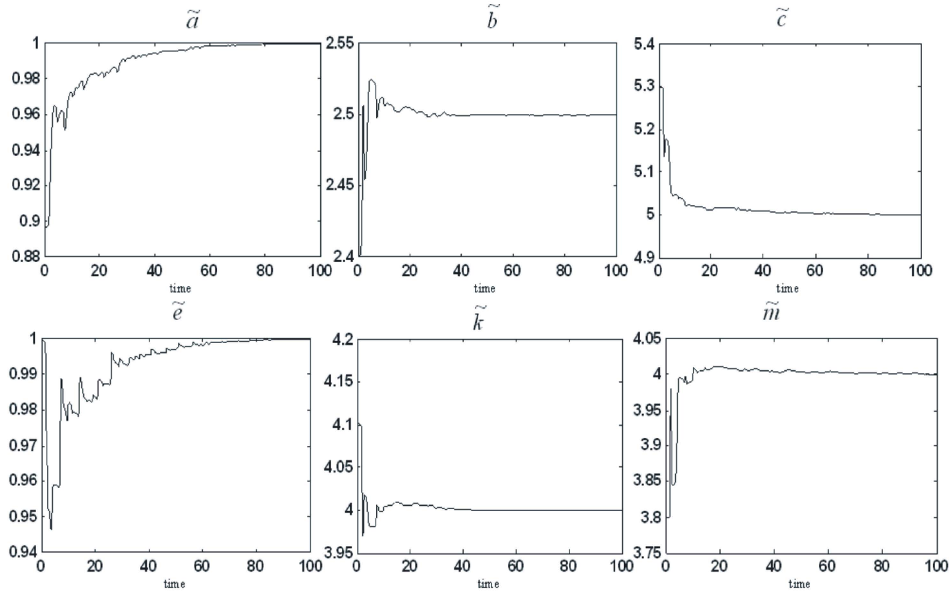

Fig. 8: Estimation of unknown parameters in slave system in the first example

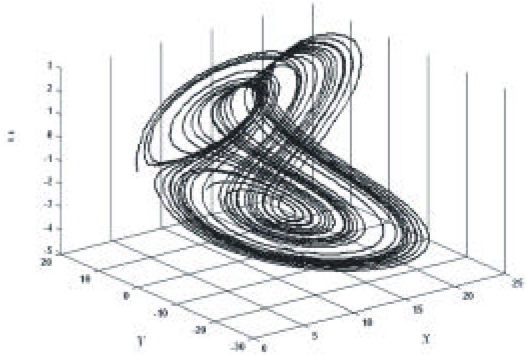

Fig. 9: Transverse butterfly-attractive of system (16) in 3-D view

Third Example: Another novel 3-D chaos system that we want to study has been introduced by C. Liu in [19]. The dynamic equation of this chaotic system is given by the following equations

$$\begin{cases} \dot{x} = a(y - x + yz) \\ \dot{y} = by - hxz \\ \dot{z} = ky - gz \end{cases} \tag{28}$$

Where *x, y, z* are the state variables and *a*, *b*, *h*, *g* and *k* are constant parameters. The behavior of the nonlinear system in (16) represents a transverse butterfly-shaped chaotic attractor [19] as shown in Fig. 9. when $a = 1$, $b = 2.5$, $k = 1$, $g = 4$ and $h = 1$ and the initial condition is (0.04, 0.2, 0). Also this chaotic system has three equilibrium points at

$$(0,0,0),\ (10,-8.633,-2.158), (10,4.633,1.158) \tag{29}$$

Which are unstable saddle focus-nodes.

Numerical example for the second 3-D systems: Here, the synchronization of two identical chaotic systems that are described by (28) is performed. For this case, the drive and response systems are taken as follows respectively

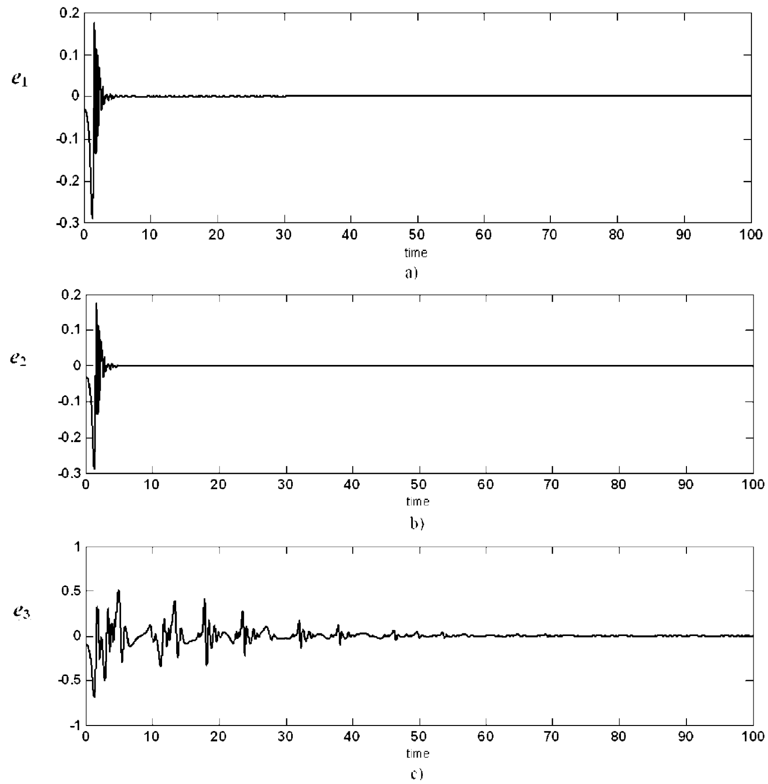

Fig. 10: Errors of the synchronization in the second example, a) e_1, b) e_2 and c) e_3

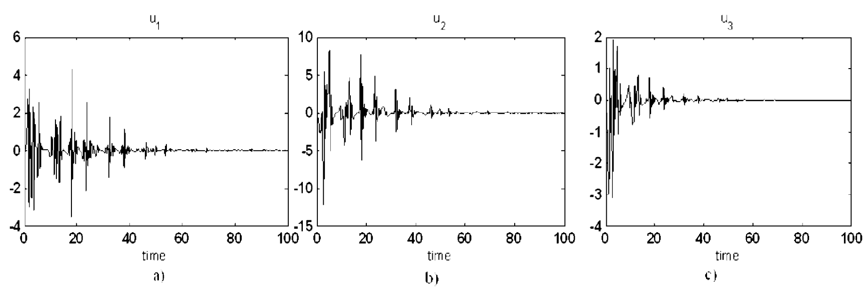

Fig. 11: Input controls in the first example, a) u_1, b) u_2 and c) u_3

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1 + y_1 z_1) \\ \dot{y}_1 = by_1 - hx_1 z_1 \\ \dot{z}_1 = ky_1 - gz_1 \end{cases} \tag{30}$$

$$\begin{cases} \dot{x}_2 = \tilde{a}(y_2 - x_2 + y_2 z_2) + u_1(t) \\ \dot{y}_2 = \tilde{b}\, y_2 - \tilde{h}\, x_2 z_2 + u_2(t) \\ \dot{z}_2 = \tilde{k}\, y_2 - \tilde{g}\, z_2 + u_3(t) \end{cases} \tag{31}$$

Where $\tilde{a}, \tilde{b}, \tilde{g}, \tilde{h}$ and $\tilde{k}$ are the estimations of a, b, g, h and k, respectively. As mentioned before, by choosing $a = 1$, $b = 2.5$, $k = 1$, $g = 4$ and $h = 1$ nonlinear system (30) shows the chaotic behavior. Also, we selected the initial condition of the drive system state as (10, –8.633, – 2.158) while the initial condition of the response system is (8.9, – 7.43333, – 2.158). Here, with respect to (1) and (2), we can define.

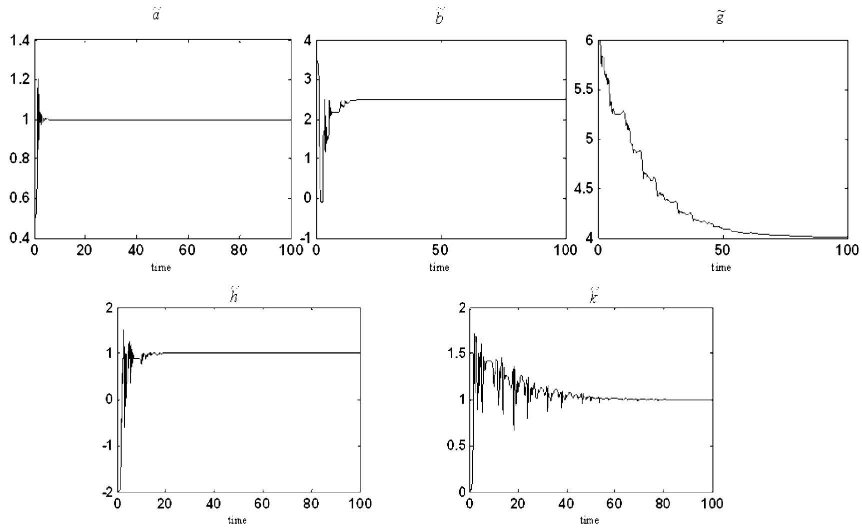

Fig. 12: Estimation of unknown parameters in slave system in the first example

$$f(x(t))=0,\ F(x(t))=\begin{bmatrix} y_1-x_1+y_1z_1 & 0 & 0 & 0 & 0 \\ 0 & y_1 & 0 & 0 & -x_1z_1 \\ 0 & 0 & -z_1 & y_1 & 0 \end{bmatrix},\ \alpha=\begin{bmatrix} a & b & g & k & h \end{bmatrix}^T \tag{32}$$

and

$$f(y(t))=0,\ F(y(t))=\begin{bmatrix} y_2-x_2+y_2z_2 & 0 & 0 & 0 & 0 \\ 0 & y_1 & 0 & 0 & -x_2z_2 \\ 0 & 0 & -z_2 & y_2 & 0 \end{bmatrix},\ \alpha=\begin{bmatrix} \tilde{a} & \tilde{b} & \tilde{g} & \tilde{k} & \tilde{h} \end{bmatrix}^T \tag{33}$$

Where $x = [x_1\ y_1\ z_1]^T$ and $y = [x_2\ y_2\ z_2]^T$. Now, by choosing K as an identity matrix

$$K=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \tag{34}$$

then the adaptive controllers and updating laws are obtained from (6) and (7) respectively as follows

$$\begin{cases} u_1(t)=\tilde{a}(e_1-e_2+y_1z_1-y_2z_2)-e_1 \\ u_2(t)=-\tilde{b}\,e_2+\tilde{h}(-x_1z_1+x_2z_2)-e_2 \\ u_3(t)=\tilde{c}e_3-\tilde{k}e_2-e_3 \end{cases} \tag{35}$$

$$\begin{cases} \dot{\tilde{a}}=x_1e_1 \\ \dot{\tilde{b}}=-y_1e_2 \\ \dot{\tilde{c}}=z_1e_3 \\ \dot{\tilde{e}}=y_1^2e_1 \\ \dot{\tilde{k}}=x_1z_1e_2 \\ \dot{\tilde{m}}=-x_1y_1e_3 \end{cases} \tag{36}$$

Where $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$ and $e_3 = z_2 - z_1$. In addition, the initial conditions for the estimation parameters were chosen as $\tilde{a}(0) = 0.5$, $\tilde{b}(0) = 3.5$, $\tilde{k}(0) = 0$, $\tilde{g}(0) = 6$ and $\tilde{h}(0) = -2$. Fig. 10 to 12 illustrate the simulation results of this example.

As it can be seen in the three examples, we succeed to achieve fully chaos synchronization between three identical even systems.

CONCLUSION

In this note, we proposed a new adaptive control method to synchronize two identical chaotic systems. Using Lyapunov stability theory, it is shown that the proposed adaptive controller with the updating law can guarantee the stabilization and therefore the synchronization action asymptotically. In order to verify the proposed method, it was also applied to the three novel chaotic systems, a 4-D system and the two 3-D systems that were reported in [17-19]. Finally, we represented the simulation results of the numerical examples.

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