

## Review and Accuracy Comparison of Soil Pressure-Sinkage Models

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**Abstract:** Soil stiffness constants govern the soil sinkage and the behavior of soil under load. To determine the soil stiffness constants in Bekker and Upadhyaya models, a sandy-loam soil reflecting general character of an agricultural soil was selected and multiplate penetration tests were conducted. For each model, from the sinkage versus pressure relationship of the soil under different loads, the average soil stiffness constants  $k_c$ ,  $k_\phi$  and  $n$  (corresponding to Bekker model) and  $k_1$ ,  $k_2$  and  $n$  (corresponding to Upadhyaya model) were determined from the sets of three sinkage tests using three small rectangular plates that differ in plate width and having nearly the same area. Tests were replicated three times for each of the three small rectangular plates. Using the models and the calculated soil stiffness constants, the pressure-sinkage behavior of a large rectangular plate was predicted. The amounts of RMSE and MRPD pertaining to Bekker model prediction were 2 mm and 11.5%, respectively. The amounts of RMSE and MRPD pertaining to Upadhyaya model prediction were 2.5 mm and 13.25 %, respectively. The results of the study showed that when the soil stiffness constants are derived from tests on the three small rectangular plates, Bekker and Upadhyaya models can be used successfully to predict the soil pressure-sinkage behavior under a large rectangular plate about three times the width, however Bekker model, to some extent, shows better results. In addition, Bekker model under predicted the sinkage values, whereas Upadhyaya model over predicted the sinkage values.

**Key words:** Soil • Pressure-sinkage • Bernstein model • Bekker model • Upadhyaya model

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### INTRODUCTION

Soil compaction continues to cause a decrease in crop yields in the world. Research throughout much of the developed world is now devoted to predicting and avoiding the effects of soil compaction. Soil compaction not only affects crop yields, but also increases energy usage to till compacted layers. Soil compaction also affects water quality when infiltration is reduced and soil erosion is thereby increased [1].

Soil compaction is a process through which pore spaces are decreased. It alters the structure of cultivated soil, i.e., the spatial arrangement, size and the shape of clods and aggregates and consequently the pore spaces inside and between these units [2]. Soil compaction can be caused by natural phenomena such as rainfall impact, soaking, internal water tension and the like. Whereas, artificial soil compaction is largely caused by soil sinkage under wheels or tracks [3].

Power, size and numbers of the tractors and agricultural machines have been increased dramatically in the last decades. This means our soil will face more compaction problem in the future than it was in the past [4, 5].

Agronomists are concerned about the effects of heavy tractors and agricultural machines on agricultural soils due to the possibility of excessive soil compaction that impedes root growth leading to yield reduction [6]. Hence, the prediction of soil sinkage under loads is an important task to determine the level of compaction in the soil.

A model that would allow agricultural engineers to manage the level of soil compaction could be most helpful if it accurately predicted situations where excessive soil sinkage could occur. Furthermore, the ability to predict soil sinkage can enable producers to till or traffic the soil when it is not in a highly compatible state or to estimate the damage being done to the soil structure due to their

excessive loading when tillage or traffic is necessary. For the last five decades, prediction of soil pressure-sinkage behavior has been of great interest to researchers in both agriculture and cross-country mobility and transport [2, 7-17]. Models presented in the literature are from a simple exponential model to an elastoplastic complicated one. Bekker and Upadhyaya models are two modified form of the exponential model which can be easily used by researchers throughout the world.

Therefore, the main objectives of this study were: a) to assess the predictability of the Bekker and Upadhyaya models under laboratory conditions and b) to compare the measured and predicted soil pressure-sinkage behavior using Bekker and Upadhyaya models.

## MATERIALS AND METHODS

**Pressure-Sinkage Models:** One of the earlier models was reported by Bernstein (1913) and Goriatchkin (1937) and the following equation was proposed to describe it [3, 17]:

$$P = kz^n \quad (1)$$

where:

- $P$  = Vertical average contact pressure, kPa
- $k$  = Soil stiffness constant for sinkage, kPa/m<sup>n</sup>
- $z$  = Depth of sinkage, m
- $n$  = A soil constant related to the soil characteristics, non-dimensional

The principal deficiency of equation 1 for prediction of soil sinkage was found to be the variability of the soil stiffness  $k$  with the size of the object on the soil. In civil engineering technology, it was known that the sinkage of the rectangular plate, at a given average vertical pressure on a particular soil, depends also on the width of the rectangle. Bekker combined the two concepts, namely the exponential pressure-sinkage relationship of equation 1 and the plate size dependence of the soil stiffness constant as follow [7, 8, 18]:

$$P = (k_c/b + k_\phi)z^n \quad (2)$$

where:

- $b$  = Plate width, m
- $k_c$  and  $k_\phi$  = Soil stiffness constants for sinkage, which are presumed to be independent of plate width, kPa/m<sup>n-1</sup> and kPa/m<sup>n</sup>, respectively

Upadhyaya proposed a modified form of the equation 1 in which the depth of sinkage was normalized by the width of the impression surface [16, 18]:

$$P = (k_1 + k_2 b)(z/b)^n \quad (3)$$

where:

- $k_1$  and  $k_2$  = soil stiffness constants for sinkage, which are presumed to be independent of plate width, kPa and kPa/m, respectively

In order to evaluate the soil stiffness constants in equations 2 and 3, it is necessary to conduct at least two soil pressure-sinkage tests using plates of different widths. The measured sets of pressure and sinkage values must then be analyzed graphically or analytically to find the best fit. From the best fit exponential curves, constants  $k$  and  $n$  can be determined for each plate of the tests. The average value of  $n$  is used together with the  $k$  values from the two plates to obtain the soil stiffness constants [18].

However, it may be risky to attempt the measurement of soil stiffness constants with tests that use only two plates, especially if they are small plates. A large variability exists in soils, even in carefully prepared laboratory samples, let alone at different locations in a field. Large rectangular plates, of the order 30 cm or more in width, can reduce the variation in experimental results, but they require large loads to approach practical sinkage pressure level and thus inconvenient and costly to perform, but smaller rectangular plates are handy for testing by one person [3].

When several plates are used and the observations are pooled to find average stiffness constants, the variation in soil stiffness constants are reduced dramatically and the measured soil stiffness constants can be used successfully to predict the pressure-sinkage behavior of a larger plate about three times the width [19]. When more than two sinkage plates are tested, a statistical method can be used to calculate the soil stiffness constants. In Bekker model, constants  $k$  and  $n$  are found for each plate. Then a graph can be made of  $k$  versus  $1/b$ , in order to solve for  $k_c$  and  $k_\phi$ . A best-fit line is found by least square analysis and  $k_c$  and  $k_\phi$  are the slope and intercept of this line [3, 8, 13, 18, 19]. In Upadhyaya model, in almost the same way, constants  $k$  and  $n$  are found for each plate. Then a graph can be made of  $k$  versus  $b$ , in order to solve for  $k_1$  and  $k_2$ . A best-fit line is found by least square analysis and  $k_1$  and  $k_2$  are the intercept and slope of this line [16, 17, 18].

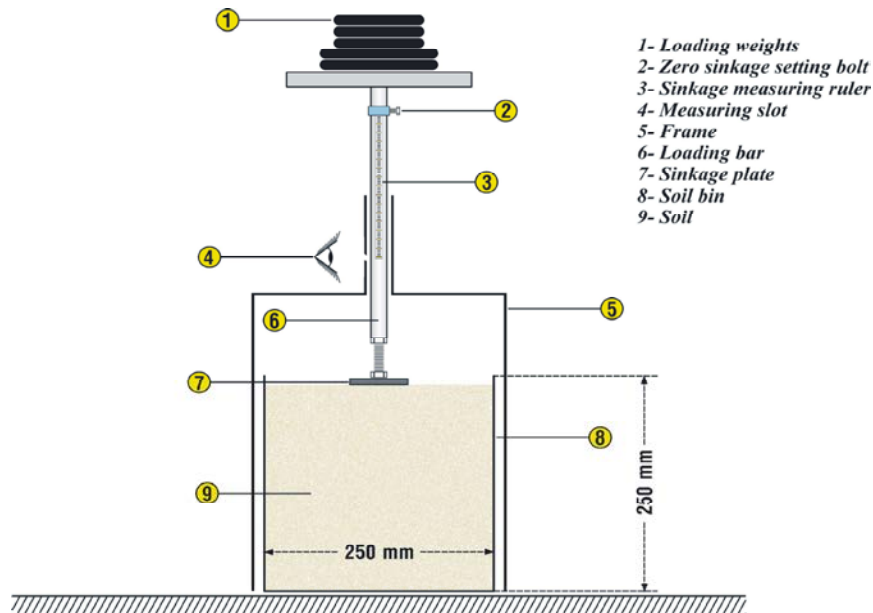


Fig. 1: Test unit

Table 1: Sizes of the three small rectangular plates used to determine the soil stiffness constants

Sinkage Plate	Width, mm	Length, mm	Aspect Ratio
1	30	83	2.8
2	33	73	2.5
3	41	61	1.5

**Test Unit Development:** A test unit was developed to determine soil stiffness constants for sinkage. A self-explanatory schematic picture of the test unit is presented in Fig. 1. Three different small rectangular plates were used in these tests. The plate dimensions are listed in Table 1. Note that the three plates have the same area, but differ in width. The aspect ratio (length/width) of these plates ranged from 1.5 to 2.8, which are similar to the ones expected for contact area of pneumatic tires (for tracks long narrow strips are recommended). The aspect ratio of a tire or track can be defined as the length of the ground contact area divided by the width.

**Experimental Procedure:** A sandy-loam soil was chosen for characterizing the agricultural soil. The sandy-loam soil was consisted of 16 % clay, 22 % silt and 62 % sand. For preparation the soil, as a first step, soil was sieved from 5 mm sieves. Then, the soil was wetted and covered with a sheet of plastic during the night in order to achieve a uniform moisture distribution. The measured soil moisture content was about 20 % (d.b.), which made the soil to be in an arable condition as in the field. The soil was leveled and then firmed in the cubic soil bin by a

wooden packer piston with the aid of a hydraulic press. In the prepared soil bin, the soil apparent bulk density was about 1650 kg/m<sup>3</sup>. For each test run, each of the three small rectangular plates was loaded slowly up to about 170 kPa and pushed downwards into the soil and at the same time the downward displacement (sinkage depth) was measured with the sinkage measuring ruler. Different loads were applied using different loading weights and tests were replicated three times for each of the three small rectangular plates.

## RESULTS

**Results of Pressure-Sinkage Tests:** The results of the pressure-sinkage tests were analyzed using the Bernstein's sinkage formula. Table 2 shows the calculated constants  $k$  and  $n$  for each of the plates and models. Very high values of coefficients of determination,  $R^2$  ranging from 0.97 to 0.98 was obtained for individual sinkage tests. However, the analysis indicated that the values of sinkage parameter  $k$  varied considerably between plates. On the other hand, the exponent  $n$  was less susceptible to this variation between plates.

As shown in Fig. 2, to obtain  $k_c$  and  $k_\phi$  by using the data from Table 2, regression analysis was applied to the constant  $k$  and the inverse of the plate width,  $1/b$ . From the linear regression results,  $k_c$  and  $k_\phi$  are the slope and the intercept of the regression line, respectively. Our attempts to relate  $k$  to  $1/b$  using equation 2 resulted in very good agreements. The calculated constants  $k_c$ ,  $k_\phi$

Table 2: Values of constants  $k$  and  $n$  for each of the plates and models

Sinkage Plate	Model					
	Bekker			Upadhyaya		
	$k$ (kPa / m <sup>n</sup> )	R <sup>2</sup>	$n$	$k$ (kPa)	R <sup>2</sup>	$n$
1	1489.4	0.98	0.6906	129.7	0.98	0.7016
2	1507.5	0.97	0.6956	145.2	0.98	0.7211
3	1646.0	0.98	0.7384	158.3	0.98	0.7328

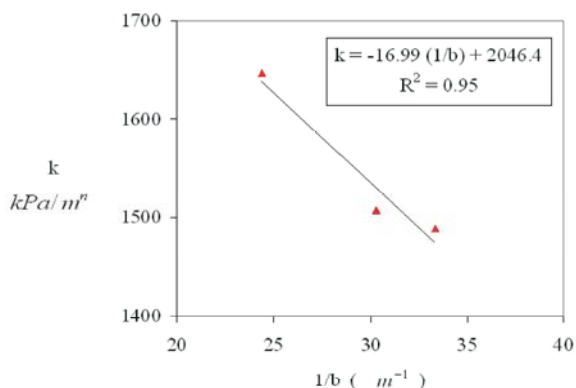


Fig. 2: Determination of  $k_c$  and  $k_\phi$  from  $k$  values of individual pressure-sinkage tests with plates of different sizes

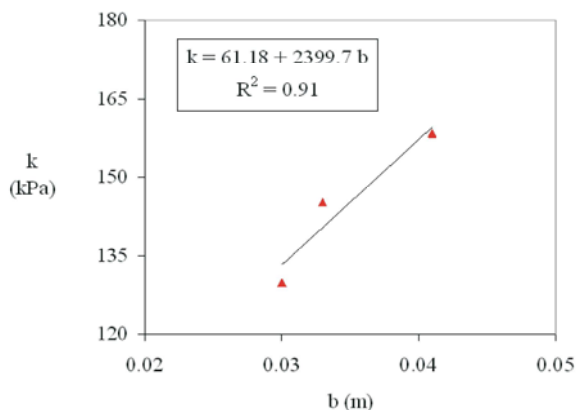


Fig. 3: Determination of  $k_1$  and  $k_2$  from  $k$  values of individual pressure-sinkage tests with plates of different sizes

and  $n$  are given in Table 3. In almost the same way, as shown in Fig. 3, to obtain  $k_1$  and  $k_2$  by using the data from Table 2, regression analysis was applied to the constant  $k$  and the plate width,  $b$ . From the linear regression results,  $k_2$  and  $k_1$  are the slope and the intercept of the regression line, respectively. Our attempts to relate  $k$  to  $b$  using equation 3 resulted in very good agreements. The calculated constants  $k_1$ ,  $k_2$  and  $n$  are given in Table 4.

Table 3: Values of constants  $k_c$ ,  $k_\phi$  and  $n$  resulted from regression analysis using three plates

Soil Stiffness Constant	$n$	$k_c$ (kPa / m <sup>n-1</sup> )	$k_\phi$ (kPa / m <sup>n</sup> )	R <sup>2</sup>
Value	0.7082	-16.99	2046.4	0.95

Table 4: Values of constants  $k_1$ ,  $k_2$  and  $n$  resulted from regression analysis using three plates

Soil Stiffness Constant	$n$	$k_1$ (kPa)	$k_2$ (kPa / m)	R <sup>2</sup>
Value	0.7185	61.18	2399.7	0.91

Table 5: Dimensions of the large rectangular plate

Sinkage Plate	Width, mm	Length, mm	Aspect Ratio
Large Rectangular	100	150	1.5

**Prediction of Soil Pressure-Sinkage Behavior for a Large Rectangular Plate:** To compare Bekker and Upadhyaya models in prediction of soil pressure-sinkage behavior, both models together with their soil stiffness constants derived from tests on the three small rectangular plates were used to predict soil pressure-sinkage behavior of a large rectangular plate and about three times the width. The dimensions of the large rectangular plate are listed in Table 5.

## DISCUSSION

**Prediction of Soil Pressure-Sinkage Behavior using Bekker Model:** Fig. 4 shows the predicted pressure-sinkage behavior of the large rectangular plate, using Bekker model and the soil stiffness constants derived from tests on the three small rectangular plates along with the experimentally measured pressure-sinkage behavior. For measuring pressure-sinkage behavior, the large rectangular plate was loaded slowly up to about 125 kPa and at the same time the sinkage depth was measured with the sinkage measuring ruler. From comparison of two curves, it could be concluded that prediction is very reasonable over the measured sinkage range, but primarily because three plates were used to enhance the level of confidence of the calculated soil stiffness constants.

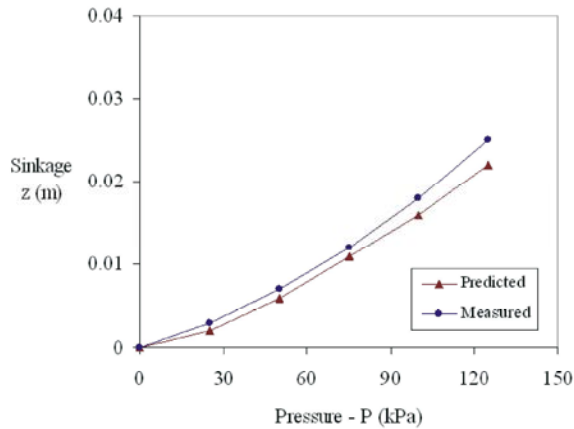


Fig. 4: Predicted pressure-sinkage behavior of the larger rectangular plate using Bekker model and the soil stiffness constant derived from the tests compared with that measured experimentally

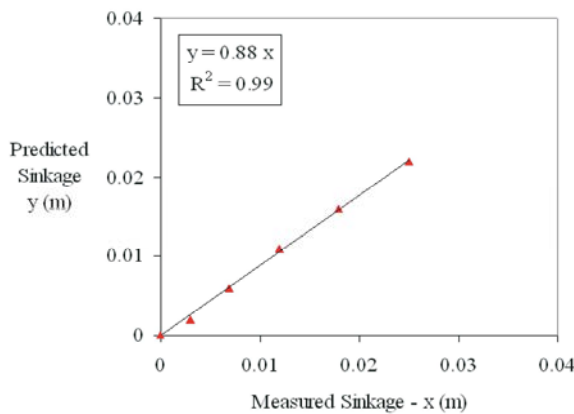


Fig. 5: Linear regression with zero intercept between sinkage values predicted using Bekker model and sinkage values measured experimentally

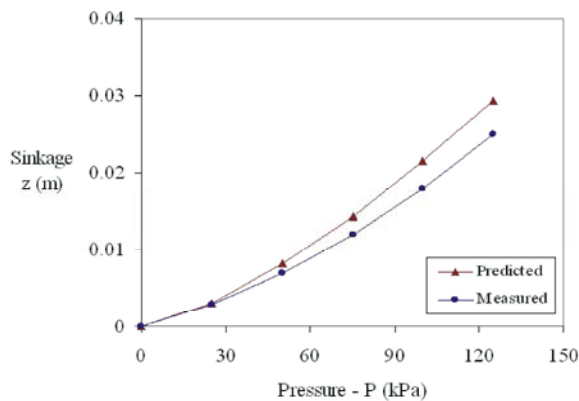


Fig. 6: Predicted pressure-sinkage behavior of the larger rectangular plate using Upadhyaya model the soil stiffness constant derived from the tests compared with that measured experimentally

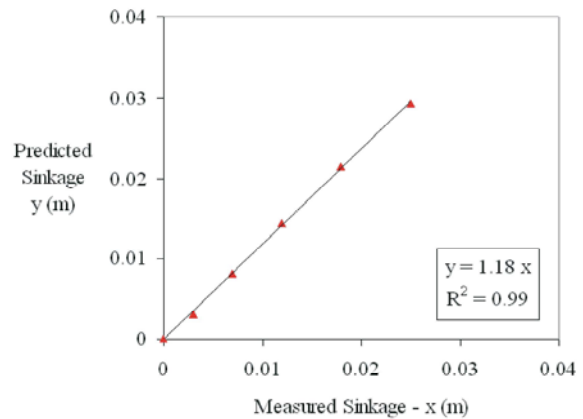


Fig. 7: Linear regression with zero intercept between sinkage values predicted using Upadhyaya model and sinkage values measured experimentally

A linear regression was performed to verify the validity of the prediction. Fig. 5 shows that the sinkage values predicted using Bekker model and the soil stiffness constants derived from tests and those measured experimentally were plotted against each other and fitted with a linear equation with zero intercept. The slope of the line of the best fit and its coefficient of determination were 0.88 and 0.99, respectively.

Root of mean square errors (RMSE) and mean relative percentage deviation (MRPD) were used to check the discrepancies between the predicted and measured results. The amounts of RMSE and MRPD were 2 mm and 11.5 %, respectively. Regarding the statistical results, the validity of the prediction was confirmed.

**Prediction of Soil Pressure-Sinkage Behavior using Upadhyaya Model:** Fig. 6 shows the predicted pressure-sinkage behavior of the large rectangular plate, using Upadhyaya model and the soil stiffness constants derived from tests on three small rectangular plates along with the experimentally measured pressure-sinkage behavior. Again, for measuring pressure-sinkage behavior, the large rectangular plate was loaded slowly up to about 125 kPa and at the same time the sinkage depth was measured with the sinkage measuring ruler. From comparison of two curves, it could be concluded that prediction is very reasonable over the measured sinkage range.

As before, a linear regression was performed to verify the validity of the prediction. Fig. 7 shows that the sinkage values predicted using Upadhyaya model and the soil stiffness constants derived from tests and those measured experimentally were plotted against each other and fitted with a linear equation with zero

intercept. The slope of the line of best fit and its coefficient of determination were 1.18 and 0.99, respectively.

Again, RMSE and MRPD were used to check the discrepancies between the predicted and measured results. The amounts of RMSE and MRPD were 2.5 mm and 13.25 %, respectively. Regarding the statistical results, the validity of the prediction was confirmed again.

The results of the study indicated that when the soil stiffness constants are derived from tests on the three small rectangular plates, Bekker and Upadhyaya models can be used successfully to predict the soil pressure-sinkage behavior under a large plate about three times the width. It should be noted that negligible discrepancies between the predicted and measured results stem out primarily from using three plates to enhance the level of confidence of the calculated soil stiffness constants. Had it been four or even five plates, the results would have been improved further. The results of the study also indicated that Bekker model, to some extent, showed better results. Furthermore, Bekker model under predicted the sinkage values, whereas the Upadhyaya model over predicted the sinkage values.

### CONCLUSION

It can be concluded that when the soil stiffness constants are derived from tests on the three small rectangular plates, Bekker and Upadhyaya models can be used successfully to predict the soil pressure-sinkage behavior under a large plate about three times the width, however Bekker model, to some extent, shows better results. Furthermore, Bekker model under predicted the sinkage values, whereas the Upadhyaya model over predicted them.

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