Agricultural Engineering Research Journal 5(3): 42-48, 2015
ISSN 2218-3906

DOI: 10.5829/idosi.aerj.2015.42.48

# Prediction of Soil Pressure-Sinkage Behavior Using Bekker Model

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**Abstract:** The prediction of soil sinkage under wheels and tracks is of great importance for determining the off-road vehicle performance and the level of compaction in the agricultural soils. Soil stiffness constants govern the soil sinkage and the behavior of soil under load. To determine the stiffness constants of soil, a sandy-loam soil reflecting general character of an agricultural soil was selected and multiplate penetration tests were conducted. Two low (1350 $kgm^{-3}$) and high (1650 $kgm^{-3}$) soil apparent densities were considered as treatments in the sandy-loam soil. For each treatment, from the pressure-sinkage relationship of soil under different loads, the average soil stiffness constants $k_C$, $k_\varphi$ and n were determined from the sets of three sinkage tests using three small rectangular plates. Tests were replicated three times for each of the three small rectangular plates. Using the determined soil stiffness constants for each treatment, the pressure-sinkage behavior of a large rectangular plate was predicted in the same soil conditions. For the low apparent density the amounts of RMSE and MRPD were 6 mm and 8%, respectively. For the high apparent density the amounts of RMSE and MRPD were 2 mm and 11.5%, respectively.

**Key words:** Soil · Compaction · Pressure-sinkage · Stiffness constants · Bekker model

## INTRODUCTION

Soil compaction is a process through which pore spaces are decreased. It alters the structure of cultivated soil, i.e. the spatial arrangement, size and the shape of clods and aggregates and consequently the pore spaces inside and between these units [1]. Soil compaction can be caused by natural phenomena such as rainfall impact, soaking, internal water tension and the like. Artificial soil compaction occurs under the downward forces of agricultural machines [2]. Soil compaction under tractors and farm machinery is of special concern because weights of these machines have been increased dramatically in the last decades and these implements create persistent subsoil compaction [3]. Agronomists are concerned about the effects of heavy tractors and agricultural machines on agricultural soils due to the possibility of excessive soil compaction that impedes root growth leading to yield reduction [4]. Hence, the prediction of soil sinkage under loads is an important task to determine the level of compaction in the soil.

For the last ten decades, prediction of soil pressure-sinkage behavior has been of great interest to researchers in both agriculture and cross-country mobility and transport [1, 5-13]. Furthermore, the ability to predict soil sinkage can enable agricultural engineers to till or traffic the soil when it is not in a highly compatible state or to estimate the damage being done to the soil structure due to their excessive loading when tillage or traffic is necessary. Models presented in the literature are from a simple exponential function to an elastoplastic complicated one. Usually, in a more complete (and thus, a more complicated) model, many parameters and variety of properties are present and have to be known prior to solving the model.

The overall objectives of this study were to measure the soil stiffness constants with tests that use three small rectangular plates and to predict the soil sinkage under a large rectangular plate using the measured soil stiffness constant in the two soil apparent densities.

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## MATERIALS AND METHODS

**Pressure-Sinkage Model:** Investigations into the nature of such phenomena and the soil parameters involved have arrived at findings in two different categories: situations in which time is considered as an important factor and those where it is not. For the case where time is not considered to be a factor, one of the earlier models was reported by Bernstein [5] and Goriatchkin [6] and the following equation was proposed to describe it [2, 12]:

$$P=kz^n \quad (1)$$

where:
P = Vertical average contact pressure, kPa
k = A soil stiffness constant for sinkage, $kPa/m^n$
z = Depth of sinkage, m
n = A soil constant related to the soil characteristics, non-dimensional

The principal deficiency of equation 1 for prediction of soil sinkage was found to be the variability of the soil stiffness *k* with the size of the object on the soil. In civil engineering technology, it was known that the sinkage of the rectangular plate, at a given average vertical pressure on a particular soil, depends also on the width of the rectangle. Bekker [7] combined the two concepts, namely the exponential pressure-sinkage relationship of equation 1 and the plate size dependence of the soil stiffness constant as follow [13, 14]:

$$P = (k_C/b+k_\varphi)z^n \quad (2)$$

where:
b = Plate width, m
$k_C$ and $k_\varphi$ = soil stiffness constants for sinkage, which are presumed to be independent of plate width, $kPa/m^{n-1}$ and $kPa/m^n$, respectively

The two parameters $k_C$ and $k_\varphi$ separate the sinkage stiffness constant k into two components. Thus, three parameters are required to describe the sinkage phenomenon. These parameters are determined using surface pressure-sinkage tests. In order to evaluate the soil constant in equation 2, it is necessary to conduct at least two soil penetration tests using plates of different widths. The measured sets of pressure and sinkage values must then be analyzed graphically or analytically to find the best fit. From the best fit exponential curves, constants k and n can be determined for each plate of the tests. The average value of n is used together with the k values from the two plates to obtain $k_C$ and $k_\varphi$ as shown below [14]:

$$k_C=b_1b_2(k_1-k_2)/(b_2-b_1) \quad (3)$$

$$k_\varphi= (k_2b_2-k_1b_1)/(b_2-b_1) \quad (4)$$

where subscripts 1 and 2 refer to the values measured for plates 1 and 2, respectively.

However, it may be risky to attempt the measurement of soil stiffness constants with tests that use only two plates, especially if they are small plates. A large variability exists in soils, even in carefully prepared laboratory samples, let alone at different locations in a field. Large rectangular plates, of the order 30 cm or more in width, can reduce the variation in experimental results, but they require large loads to approach practical sinkage pressure level and thus inconvenient and costly to perform, but smaller rectangular plates in the range of five to ten cm are handy for testing by one person.

It has been shown that the variation in $k_C$ and $k_\varphi$ can be considerable when only two small plates are used. When several plates are used rather than two and the observations are pooled to find average stiffness constants, then the variation in $k_C$ and $k_\varphi$ are reduced dramatically. When more than two sinkage plates are tested, a statistical method can be used to calculate the stiffness constants. Constants k and n are found for each plate. Then a graph can be made of k versus 1/b, in order to solve for $k_C$ and $k_\varphi$. A best-fit line is found by least square analysis and $k_C$ and $k_\varphi$ are the slope and intercept of this line [15].

**Test Unit Development:** A test unit was developed to determine soil stiffness constants for sinkage. A self-explanatory schematic picture of the test unit is presented in Fig. 1. Three different rectangular plates were used in these tests. The plate dimensions are listed in Table 1. Note that the three plates have the same contact area, but differ in width only. The aspect ratio (length/width) of these plates ranged from 1.5 to 2.8, which are similar to the ones expected for pneumatic tires contact areas (for tracks long narrow strips are recommended). The aspect ratio of a tire or track footing can be defined as the length of the ground contact area divided by the width.

**Experimental Procedure:** A sandy-loam soil was chosen for characterizing the agricultural soil. The sandy-loam soil was consisted of 16% clay, 22% silt and 62% sand.

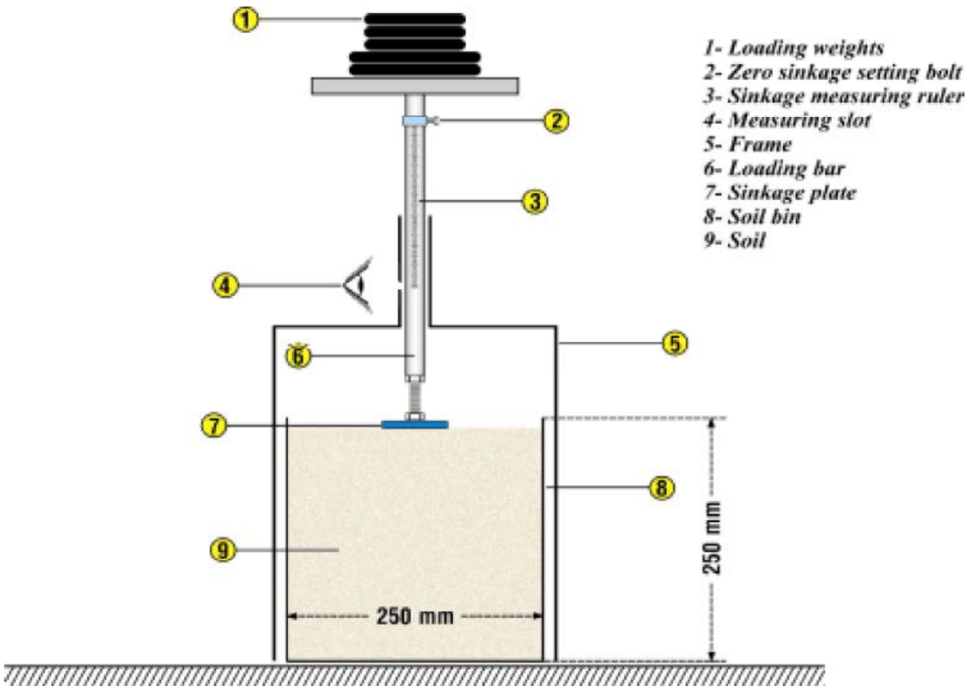


Fig. 1: Test unit

Table 1: Sizes of sinkage plates used to determine soil stiffness constants

| Sinkage Plate No. | Width, mm | Length, mm | Aspect Ratio |
|---|---|---|---|
| 1 | 30 | 83 | 2.8 |
| 2 | 33 | 73 | 2.5 |
| 3 | 41 | 61 | 1.5 |

To prepare soil samples, as a first step, soil was sieved using a mesh size of 5 mm. Then, the soil was watered and covered with a sheet of plastic during the night in order to achieve a uniform moisture distribution. The measured soil moisture content was about 20% (dry basis), which made the soil sample to be in an arable condition as in the field. The soil was leveled and then firmed in the cubic soil bin by a wooden packer piston with the aid of a hydraulic press.

Two low (1350 $kgm^{-3}$) and high (1650 $kgm^{-3}$) apparent densities, representing the field apparent densities, were considered as treatments. For each test run, each of three small rectangular plates was loaded slowly up to about 170 kPa and pushed downwards into the soil and at the same time the downward displacement (sinkage depth) was measured with the sinkage measuring ruler. Different loads were applied using different loading weights and tests were replicated three times for each of the three small rectangular plates in both apparent densities.

## RESULTS AND DISCUSSION

**Pressure-Sinkage Tests Results:** The results of the pressure-sinkage tests were firstly analyzed using the Bernstein model. Table 2 shows the calculated constants k and n for each of the plates and treatments. Very high values of coefficients of determination, $R^2$ ranging from 0.90 to 0.99 were obtained for individual sinkage tests. However, the analysis indicated that the values of sinkage parameter k varied considerably between plates. On the other hand, the exponent n was less susceptible to this variation between plates. As shown in Fig. 2 and Fig. 3, to obtain $k_C$ and $k_\varphi$ by using the data from Table 1, regression analysis was applied to the constant k and the inverse of the plate width, 1/b. From the linear regression results, $k_C$ and $k_\varphi$ are the slope and the intercept of the regression line, respectively. Our attempts to relate k to 1/b using equation 2 resulted in very good agreements. The calculated constants $k_C$ and $k_\varphi$ for each treatment are given in Table 3.

**Prediction of Soil Sinkage under a Large Rectangular Plate:** Since footing problems have been already used to check the validity of the Bekker model [13, 15], the soil stiffness constants measured with three small rectangular plates in two soil apparent densities were used to predict soil sinkage under a large rectangular plate with dimensions listed in Table 4 in the same soil conditions.

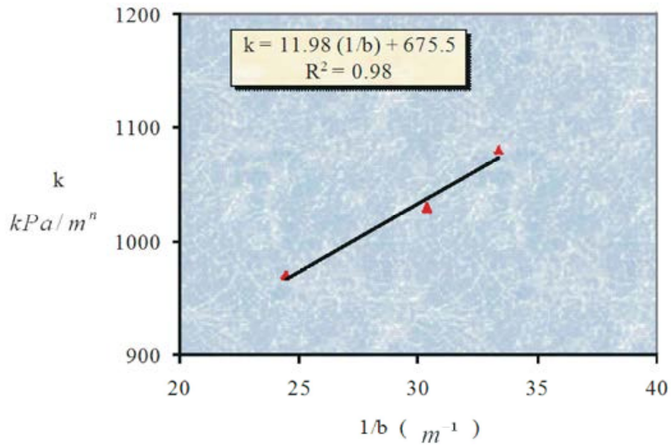


Fig. 2: Determination of $k_C$ and $k_\varphi$ from k values of individual sinkage tests with plates of different sizes in low soil apparent density

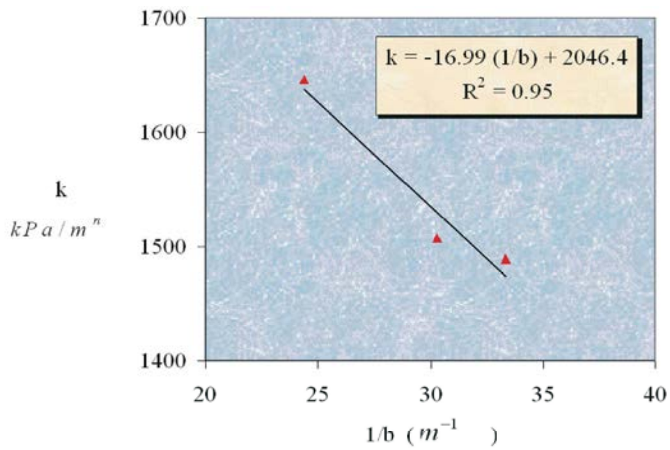


Fig. 3: Determination of $k_C$ and $k_\varphi$ from k values of individual sinkage tests with plates of different sizes in high soil apparent density

Table 2: Values of constants k and n for each plates and treatments

| | Treatment | | | | | |
|---|---|---|---|---|---|---|
| | Low soil apparent density | | | High soil apparent density | | |
| Sinkage Plate No. | k (kPa/$m^n$) | $R^2$ | n | k (kPa/$m^n$) | $R^2$ | n |
| 1 | 1080.8 | 0.95 | 0.8332 | 1489.4 | 0.89 | 0.6906 |
| 2 | 1029.6 | 0.90 | 0.8243 | 1507.5 | 0.97 | 0.6956 |
| 3 | 970.80 | 0.94 | 0.7883 | 1646.0 | 0.99 | 0.7384 |

Table 3: Results of regression analysis of low and high soil apparent densities using three rectangular sinkage plates

| Soil | n | $k_C$ (kPa/$m^{n1}$) | $k_\varphi$ (kPa/$m^n$) | $R^2$ |
|---|---|---|---|---|
| Low apparent density | 0.8153 | 11.98 | 675.50 | 0.98 |
| High apparent density | 0.7082 | -16.99 | 2046.4 | 0.95 |

Table 4: Sizes of the large rectangular plate

| Sinkage Plate | Width, mm | Length, mm | Aspect Ratio |
|---|---|---|---|
| Large Rectangular | 100 | 150 | 1.5 |

**Low Apparent Density:** Fig. 4 shows the predicted pressure-sinkage behavior of soil under the large rectangular plate, using the soil stiffness constants derived from tests on three small rectangular plates on the soil with a low apparent density along with the measured values. For measuring pressure-sinkage behavior, the larger rectangular plate was loaded slowly up to about 125 kPa and at the same time the sinkage depth was measured

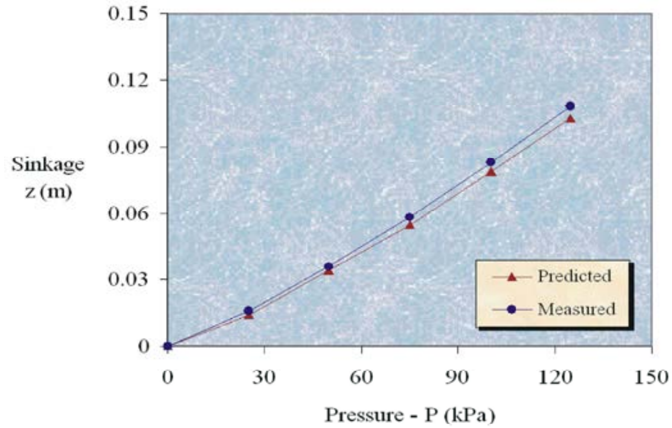


Fig. 4: Predicted pressure-sinkage behavior of the larger rectangular plate compared with that measured experimentally on low soil apparent density

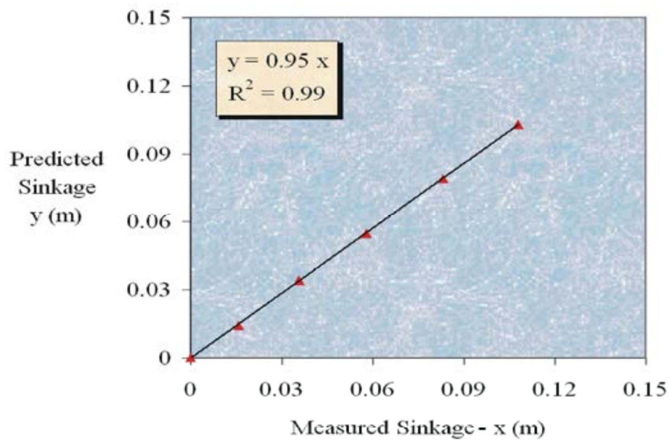


Fig. 5: Predicted and measured sinkage values on low soil apparent density

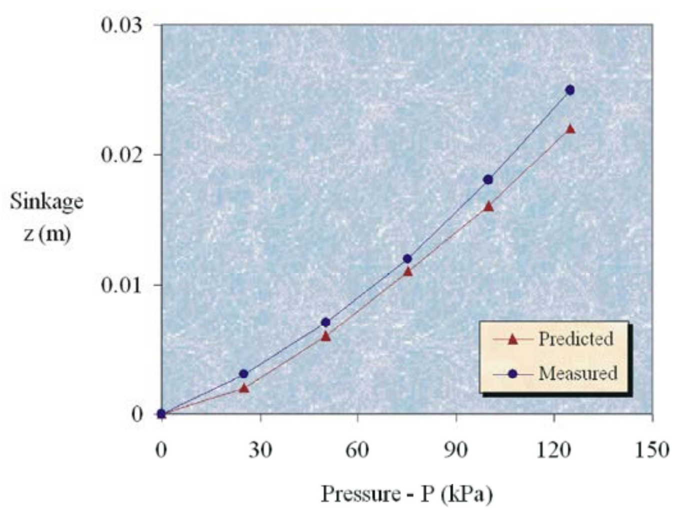


Fig. 6: Predicted pressure-sinkage behavior of the larger rectangular plate compared with that measured experimentally on high soil apparent density

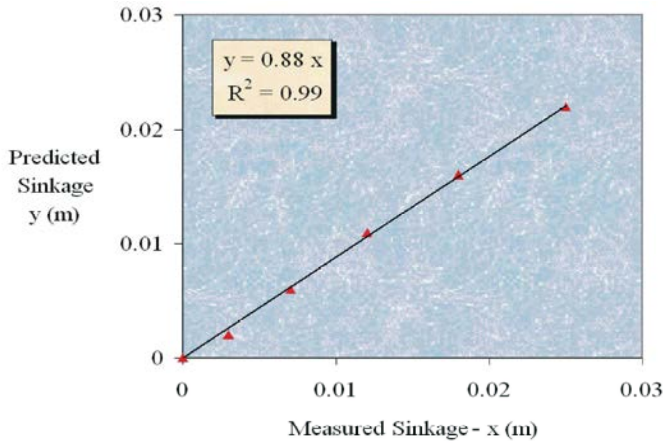


Fig. 7: Predicted and measured sinkage values on high soil apparent density

with the sinkage measuring ruler. From comparison of two curves, it could be concluded that prediction is very reasonable over the measured sinkage range. A linear regression was performed to verify the validity of the prediction. Fig. 5 shows that the sinkage values predicted using the soil stiffness constants derived from tests and those measured experimentally were plotted against each other and fitted with a linear equation with zero intercept. The slope of the line of the best fit and its coefficient of determination were 0.95 and 0.99, respectively. Root of mean square errors (RMSE) and mean relative percentage deviation (MRPD) were used to check the discrepancies between the predicted and measured results. The amounts of RMSE and MRPD were 6 mm and 8%, respectively. Regarding the statistical results, the validity of the prediction was confirmed.

**High Apparent Density:** Fig. 6 shows the predicted pressure-sinkage behavior of soil under the larger rectangular plate, using the soil stiffness constants derived from tests on three small rectangular plates on the soil with a high apparent density along with the measured values. Again, for measuring pressure-sinkage behavior, the larger rectangular plate was loaded slowly up to about 125 kPa and at the same time the sinkage depth was measured with the sinkage measuring ruler. From comparison of two curves, it could be concluded that prediction is very reasonable over the measured sinkage range. As before, a linear regression was performed to verify the validity of the prediction. Fig. 7 shows that the sinkage values predicted using the soil stiffness constants derived from tests and those measured experimentally were plotted against each other and fitted with a linear equation with zero intercept. The slope of the line of best fit and its coefficient of determination were 0.88 and 0.99, respectively. Again, root of mean square errors (RMSE) and mean relative percentage deviation (MRPD) were used to check the discrepancies between the predicted and measured results. The amounts of RMSE and MRPD were 2 mm and 11.5%, respectively. Regarding the statistical results, the validity of the prediction was confirmed again. More likely reason for such negligible discrepancies between the predicted and measured results using the soil stiffness constants stem out primarily from using three plates to enhance the level of confidence of the calculated soil stiffness constants. Had it been four or even five plates, the results would have been improved further [15].

## CONCLUSIONS

The soil stiffness constants measured with three small rectangular plates were used to predict the pressure-sinkage behavior of soil under a large rectangular plate using Bekker model. The statistical results of study confirmed the validity of the prediction and demonstrated that it could be very useful to predict the soil sinkage under tires and tracks of tractors and agricultural machines.

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