

Thermal Radiation and Heat Transfer Effects on the Steady MHD Fluid Flow past a Vertical Porous Plate with Injection

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Abstract: The boundary layer steady flow and heat transfer towards a porous plate in presence of a magnetic field and injection is presented in this analysis. Thermal radiation and heat absorption terms are incorporated in the temperature equation. Similarity transformations are used to convert the partial differential equations corresponding to the momentum and energy equations into non-linear ordinary differential equations. Numerical solutions of these equations are obtained by shooting technique.

Key words: Porous plate • MHD • Thermal radiation • Heat absorption • Similarity solutions • Injection

INTRODUCTION

The study of fluid flow and heat transfer over a porous medium is a subject of interest for many researchers working in the area of the two-dimensional flows. The reasons lie in the fact that such kinds of investigations have applications in the manufacturing industry (For example hot rolling, continuous casting, extrusion process, manufacturing of sheets and coating). Further, the flow in a porous media is used to study the migration of underground water, movement of oil, gas and water through the reservoir, water purification and ceramic engineering. The initial work was investigated by Sakiadis [1] for two-dimensional boundary layer flow when the plate is moving with constant velocity. The Sakiadis's problem for heat transfer analysis was studied by Erickson *et al.* [2]. In the above mention studies the velocity of the sheet is assumed to be constant. This assumption of constant velocity is adequate when we are interested in the analysis of continuous extrusion of polymer sheets. Due to the flexibility of polymer materials, a stretching may occur. Crane [3] found a closed form solution for boundary layer flow by imposing the condition of stretching wall. In many real situations the thermal buoyancy also plays part for the occurrence of fluid flow. The buoyancy forces occur due to the heating and cooling of stretching sheets resulting in producing

changes to both flow and temperature fields as discussed by Chen and Strobel [4]. The literature survey indicates that work in this direction is carried out by many workers in the field [5] to [8] and reference therein. On the other hand, the constitutive relationships for rate type fluids are implicit and elimination of stress components from the equation of motion is not straightforward. This fact is the major cause for the lack of literature on the two-dimensional flow of rate type fluids. Recently, the effect of thermal radiation on the steady laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet was reported by Bidin and Nazar [9]. Of late, El-Aziz [10] and Ishak [11] described the flow and heat transfer past an exponentially stretching sheet. The analysis of ow of viscoelastic uids in the absence of a magnetic field was studied by Siddappa and Khapate [20], Siddappa and Abel [19].

The radiation heat transfer effects on different flows are very important in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids,

MHD accelerators, power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive gray fluids. Gupta and Gupta [18] analyzed the momentum, heat and mass transfer in the boundary layer over a stretching sheet subject to suction or blowing. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection laws studied by Chen *et al.* [12]. Badruddin *et al.* [13] analyzed the free convection and radiation characteristics for a vertical plate embedded in a porous medium. Madhusudhana Rao B, Viswanatha reddy G and Raju M.C [14] studied MHD transient free convection and chemically reactive flow past a porous vertical plate with radiation and temperature gradient dependent heat source in slip flow regime. Takhar *et al.* [15] investigated the radiation effects on MHD free convection flow of a radiating gas past a semi-infinite vertical plate. Muthukumaraswamy and Ganesan [16] studied the diffusion and first-order chemical reaction on impulsively started infinite vertical plate with variable temperature. Swathi Mukhopadhyay [17] studied the slip effects on MHD boundary layer flow over exponential stretching sheet with thermal radiation.

The partial differential equations governing the flow have been transferred into a system of ordinary differential equations using similarity transformations and solved numerically using Runge-kutta method with shooting technique. Numerical calculations are taken up to desire level of accuracy were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically.

The analysis of the results obtained shows that the flow field is influenced appreciably by the slip parameter in the presence of magnetic field and injection at the wall. Estimation of skin friction which is very important from the industrial application point of view is also presented in this analysis. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

Formulation of the Problem: Consider a two dimensional incompressible viscous electrically conducting fluid past a porous plate with injection. In rectangular Cartesian coordinate system, we take x-axis along the plate in the direction of flow and y-axis normal to it. The surface of the plate is maintained at a uniform temperature T_w . The flow is confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed.

Governing equations describing the conservation of mass, momentum and energy equations are given by;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B^2}{\rho} + \frac{\gamma}{k'} \right) u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) \tag{3}$$

with the boundary conditions;

$$\begin{aligned} u = b, \quad v = v_1, \quad T = T_w \quad \text{at } y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \tag{4}$$

where γ is kinematic viscosity and we consider the following dimensionless similarity transformations;

$$\begin{aligned} \eta = y \sqrt{\frac{b}{\gamma x}}, \quad \psi = \sqrt{b \gamma x} f(\eta), \quad T = T_\infty + \theta(T_w - T_\infty), \\ M = \frac{2x\sigma B^2}{b\rho}, \quad K = \frac{2\gamma x}{K'b^2}, \quad Pr = \frac{\gamma \rho C_p}{k}, \quad R = \frac{\gamma x Q_0}{k b(T_w - T_\infty)}, \quad F_A = -2v_1 \sqrt{\frac{x}{b\gamma}} \end{aligned} \tag{5}$$

where $f(\eta)$ is dimensionless stream function. In the view of equation (5), the equations (2) and (3) are reduced to the following ordinary non linear differential equations;

$$2f''' + ff'' - (M + K)f' = 0 \tag{6}$$

$$2\theta'' + Pr f \theta' + Q\theta = 0 \tag{7}$$

and the corresponding boundary conditions in non-dimensional quantities are given by,

$$\begin{aligned} f' = 1, \quad f = F_A, \quad \theta = 1 \quad \text{at } \eta = 0 \\ f' \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \end{aligned} \tag{8}$$

Numerical Solution: The set of non linear ordinary differential equations (6) and (7) together with the boundary conditions (8) is solved by fourth-order Runge-Kutta method along with Shooting technique for the prescribed parameters Magnetic parameter (M), Permeability of porous medium (K), Prandtl number (Pr), Radiation parameter (R) and Injection parameter (F_w).

In the boundary conditions (8), there are four asymptotic boundary conditions and hence there are four unknown surface conditions $f'(0)$, $f(0)$, $\theta(0)$. Values of these unknown surface conditions are obtained by shooting technique. A program is set up for the above mentioned procedure along with the Runge-kutta method to solve the equations (6) and (7) with the boundary conditions (8). A step size of $\Delta\eta = 0.01$ was chosen and value of η_w was found to each iteration loop by $\eta = \eta + \Delta\eta$.

RESULTS AND DISCUSSION

The dimensional less velocity and temperature profiles for different value of Magnetic parameter M with $K=1$, $Pr=0.71$, $Q=0.3$, $F_A=1$ are shown in Fig. 1 and Fig. 2. As M increases, velocity found to decrease (Fig. 1) and the temperature increases (Fig. 2). The transverse magnetic field opposes the motion of the fluid and the rate of transport is reduced considerably. This is because with the increase in M, Lorentz force increases and it produces more resistance to the fluid flow. In addition with this the thermal boundary layer thickness also increases as M increases.

The influence of permeability of porous medium K with $M=1$, $Pr=0.71$, $Q=0.3$, $F_A=1$ on velocity and temperature profiles is represented in Fig. 3 and Fig. 4 respectively. As K increases velocity decreases (Fig. 3) and temperature increases (Fig. 4). In other words this increase of permeability of porous medium decreases the thickness of momentum boundary layer which eventually increases the heat transfer.

The effects of Radiation parameter Q with $K=1$, $Pr=0.71$, $M=1$, $F_A=1$ on the temperature are represented in Fig. 5 and it is observed that heat transfer increases for increasing value of Q. It is noticed from Fig. 6 (with $K=1$, $Q=0.3$, $M=1$, $F_A=1$) the temperature decreases as Pr increases and this is due to fact that thermal boundary layer decreases with an increase in Pr. When $Pr > 1.0$ momentum disturbance propagates farther into the free stream than a thermal disturbance. The reverse is true for $Pr < 1.0$. This effect is due to the large temperature gradient that occurs because of the thin thermal boundary layer.

The velocity component for different values of injection parameter F_w with $K=1$, $Pr=0.71$, $M=1$, $Q=0.3$ is portrayed in Fig. 7. It is observed that for increasing value of F_w the velocity within the boundary layer exponentially decreases. Due to wall injection ($F_w > 0$), resistance of the fluid increases and has a behavior to reduce the velocity of the fluid flow, which is observed from Fig. 7. But the wall injection ($F_w < 0$) produces the reverse effect. The

temperature profiles for the various values of F_w with $K=1$, $Pr=0.71$, $M=1$, $Q=0.3$ are shown in Fig. 8. Similar to velocity of the fluid wall injection ($F_w > 0$) has a tendency to reduce the thermal boundary layer thickness, which can be shown in Fig. 8. But when $F_w < 0$, thermal boundary layer thickness increases. Because of this fact, the temperature within the thermal boundary layer gradually decreases as F_w increases.

From the Table 1, it is seen that skin friction coefficient increases with the increasing values of K, M, F_A and no effect of Pr and Q is observed on skin friction. The rate of heat transfer coefficient is increases with an increase in F_A and Pr, but the opposite trend is observed with an increase in K, M and Q.

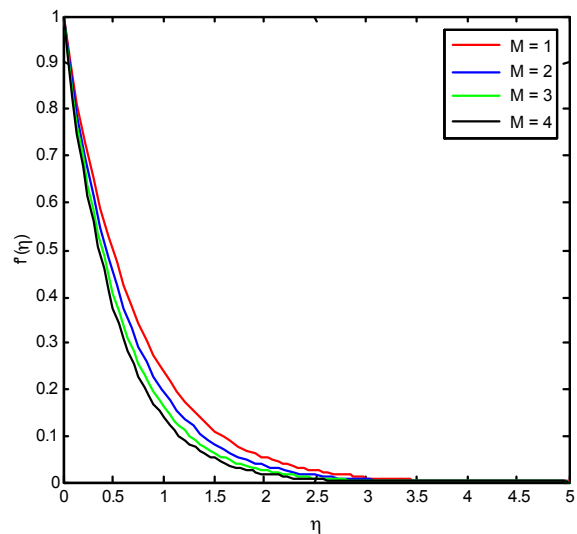


Fig. 1: Effects of M on Velocity

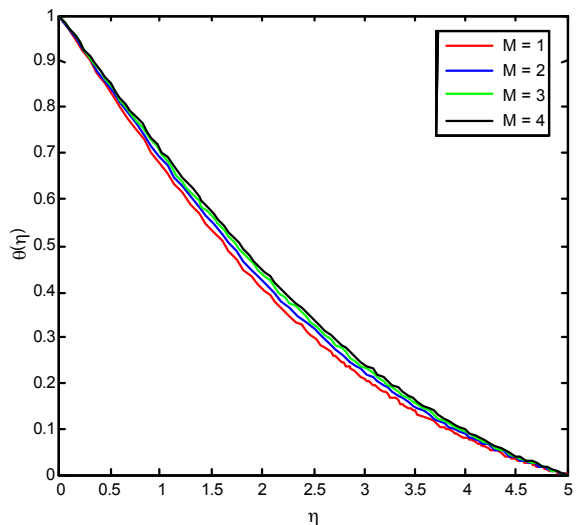


Fig. 2: Effects of M on Temperature

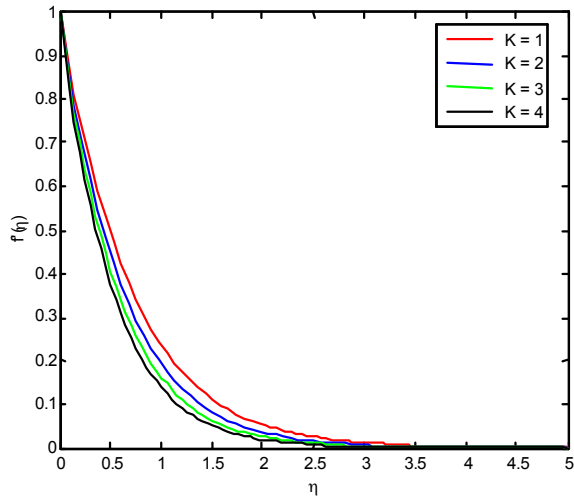


Fig. 3: Effects of K on Velocity

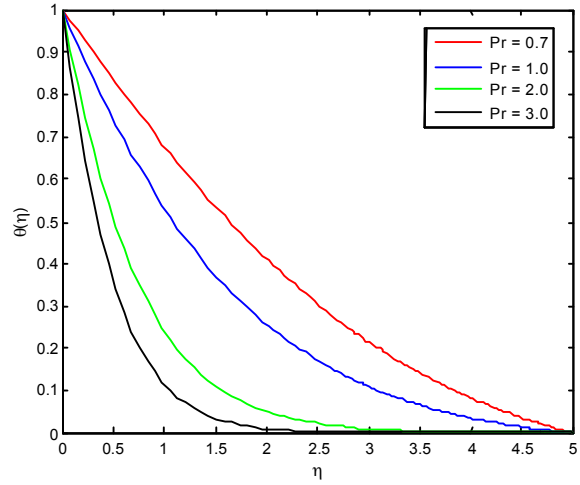


Fig. 6: Effects of Pr on Temperature

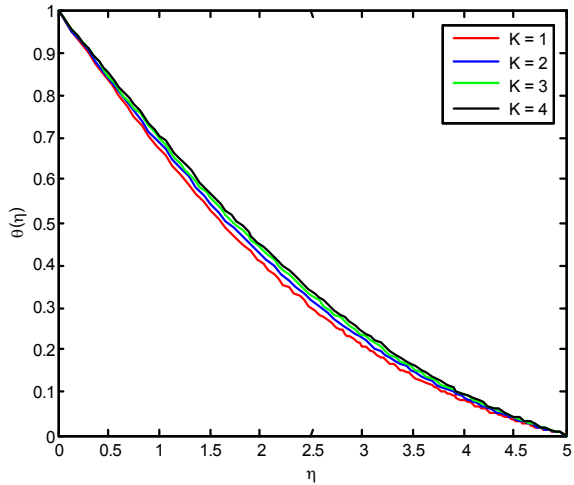


Fig. 4: Effects of K on Temperature

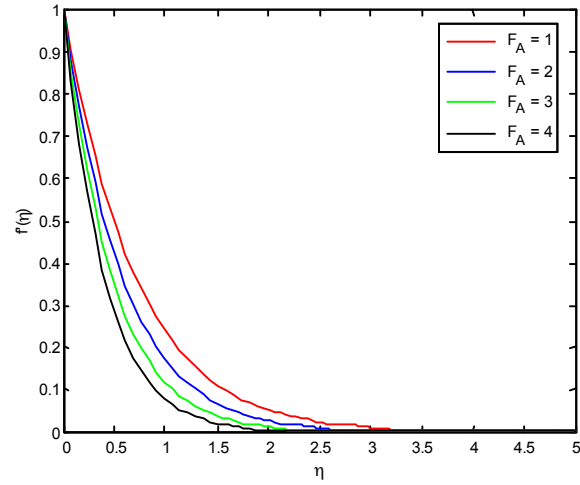


Fig. 7: Effects of F_A on Velocity

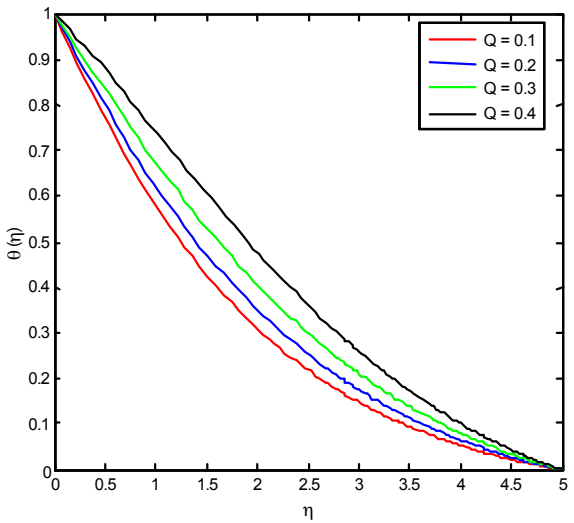


Fig. 5: Effects of Q on Temperature

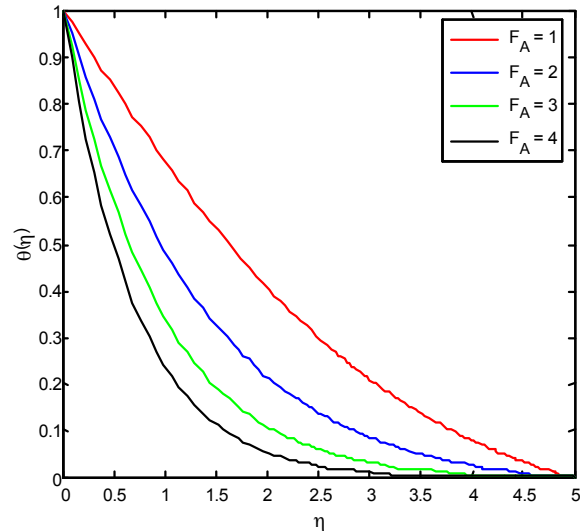


Fig. 8: Effects of F_A on Temperature

Table 1: Numerical values of $-f'(0)$ and $-\theta(0)$ for various values of the flow parameters

K	M	F_A	Pr	Q	$-f'(0)$	$-\theta(0)$
0.2	0.5	2.0	0.5	0.3	1.415061	0.790758
0.4	0.5	2.0	0.5	0.3	1.465468	0.788593
0.6	0.5	2.0	0.5	0.3	1.513776	0.786565
0.1	1.0	2.0	0.5	0.3	1.513776	0.786565
0.1	2.0	2.0	0.5	0.3	1.730259	0.778046
0.1	3.0	2.0	0.5	0.3	1.916330	0.771444
0.1	0.5	0.5	0.5	0.3	1.021964	0.519331
0.1	0.5	1.0	0.5	0.3	1.388993	0.791897
0.1	0.5	2.0	0.5	0.3	1.800953	1.101657
0.1	0.5	2.0	0.1	0.3	1.388993	0.234649
0.1	0.5	2.0	0.3	0.3	1.388993	0.392706
0.1	0.5	2.0	0.7	0.3	1.388993	0.781366
0.1	0.5	2.0	0.5	0.2	1.388993	0.835657
0.1	0.5	2.0	0.5	0.4	1.388993	0.745737
0.1	0.5	2.0	0.5	0.6	1.388993	0.644924

CONCLUSIONS

In the present investigation, the influence of various flow parameters on the fluid velocity and temperature is illustrated and discussed. The results give a view towards the stagnation flow of MHD fluid in the presence of magnetic field, injection and thermal radiation parameters. Based on the present investigation the following observations are made.

The velocity decreases with the increasing values of Magnetic parameter whereas temperature increases. The temperature increases with an increase in Radiation parameter. An increase in the injection parameter leads to decrease in the fluid velocity. This is due to the fact that the suction stabilizes the boundary layer growth and the temperature within the thermal boundary layer gradually decreases as suction parameter increases. The temperature decreases when Prandtl number increases. This is due to the fact that the fluid with higher Prandtl number has relatively low thermal conductivity. An increase of permeability of porous medium decreases the thickness of momentum boundary layer which eventually increases the heat transfer. The transverse magnetic field opposes the motion of the fluid and the rate of transport is reduced considerably. The thermal boundary layer thickness also increases as M increases.

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