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Stability Analysis of Predatorprey Model with Ratio-Dependent Functional Response

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Abstract: This paper concerns with a two dimensional nonlinear dynamical predator-prey model with ratio-dependent functional response. Dynamical analysis involving determination of equilibrium points on their local stabilities is presented.

Key words: Predator-prey · Ratio-dependent · Equilibrium points · Local stability

INTRODUCTION

Predator-prey behavior is a form of very common biological interaction in nature. Mathematical model for predator-prey interaction is studied originally by Lotka [1] and Volterra [2] and is known as Lotka-Volterra model. The model is only consider four factors such as growth rate of prey, predation rate, mortality rate of predator and conversion rate to change prey biomass into predator reproduction. Notice that all of the rates are linear. However, in the real life, predator-prey interaction does not depend only on those factors. Therefore, much developments of the model are proposed based on biological assumptions in the real life.

According to some biologists, such as Arditi and Ginzburg [3], ecological functional response should depend on the density of prey and predator, since predators occasionally have to search and compete for the prey. One of the functional responses which depend on the density of prey and predator is ratio-dependent functional response (see Xiao and Ruan [4], Edwin [5]). Therefore, in this paper we concern with dynamical analysis of predator-prey model with ratio-dependent response function. It is assumed that prey as well as predator grows logistically, since predator has other food source besides prey. Hence, the predator has two growth rate, namely logistic and predation growth. In order to control the amount of predator population, it is assumed that a linear rate of harvesting is applied to predator population.

The Model: Predator-prey model in this paper modifies the model discussed by Kar and Chaudhuri [6] by replacing Holling type II functional response by ratio-dependent functional response. Hence, the model that we concern with is;

$$\begin{cases} \dfrac{dX}{dt} = r\left(1-\dfrac{X}{K_1}\right)X - \dfrac{aXY}{Y+bX} \\ \dfrac{dY}{dt} = s\left(1-\dfrac{Y}{K_2}\right)Y + \dfrac{cXY}{Y+bX} \end{cases} \quad (1)$$

where X represents prey density, Y is predator density, r and s are growth rate of prey and predator respectively, K_1 and K_2 represent carrying capacity of prey and predator respectively, a is parameter of capturing rate predator on prey, $1/b$ is Michaelis-Menten constant and c represents conversion rate to change prey biomass into predator reproduction.

Equilibria: The possible equilibrium points of system (1) are E_1 $(0, K_2)$, E_2 $(K_1, 0)$ and E_3 (X^*, Y^*) where

$$X^* = \frac{-B+\sqrt{B^2-4AC}}{2A} \quad \text{or} \quad X^* = \frac{-B-\sqrt{B^2-4AC}}{2A} \quad \text{and}$$

$$Y^* = \frac{rbX^*\left(1-\dfrac{X^*}{K_1}\right)}{a-r\left(1-\dfrac{X^*}{K_1}\right)}.$$

Here, $A = \dfrac{r}{K_1}\left(\dfrac{sb}{K_2}+\dfrac{cr}{abK_1}\right)$,

$$B = \frac{r}{K_1}\left(s+\frac{2c}{ab}(a-r)\right)-\frac{rsb}{K_2},$$

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$C = (a-r)\left(s + \frac{c}{ab}(a-r)\right)$, and

$$D = \left(\frac{rs}{K_1} - \frac{rsb}{K_2}\right) - \frac{rrsb}{K_1K_2}(a-r)\left(s+\frac{c}{b}\right)$$

Proposition 1: Equilibriumpoint E3(X*, Y*) exists if one of the following conditions satisfied.

$$r > a, \tag{2}$$

or

$$r < a \text{ , B< 0 and D>0,} \tag{3}$$

or

$$r = a \text{ and } 1 < \frac{K_1 b}{K_2} \tag{4}$$

Remark: X^* and Y^* satisfies the following equations.

$$r\left(1 - \frac{X^*}{K_1}\right) = \frac{aY^*}{Y^* + bX^*} \tag{5}$$

and

$$r\left(1 - \frac{Y^*}{K_2}\right) = -\frac{cX^*}{Y^* + bX^*} \tag{6}$$

Local Stability Analysis: Stability of equilibrium points is investigated by doing linearization on thesystem (1) around each equilibrium points.

Theorem 4.1: The equilibrium point E_1 (0, K_2) is locally asymptotically stable if $r < a$.

Proof: At E_1 (0, K_2), the Jacobean matrix becomes

$$J(E_1) = \begin{pmatrix} r-a & 0 \\ c & -s \end{pmatrix}$$

Jacobian matrix of E_1 has negativeEigen value for $r < a$. Hence E_1 (0, K_2) is locally asymptotically stable if $r < a$ and unstable if $r > a$.

Theorem 4.2: The equilibrium point E_2 (K_1, 0) is unstable.

Proof: At E_2 (K_1, 0), the Jacobean matrix is

$$J(E_2) = \begin{pmatrix} -r & -\frac{a}{b} \\ 0 & s+\frac{c}{b} \end{pmatrix}$$

Since $s + \frac{c}{b} > 0$, equilibrium point $E_2$is unstable.

Theorem 4.3: The equilibrium point E_3 (X^*, Y^*) is locally asymptotically stable provided $\Delta > 0$ and

$\Gamma < 0$ Where $\Delta = xw - yz$ and $1 = x + w$.

Proof: AtE_3 (X^*, Y^*), the Jacobean matrix becomes

$$J(E_3) = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

where $x = r\left(1 - \frac{2X^*}{K_1}\right) - a\left(\frac{Y^*}{Y^* + bX^*}\right)^2$,

$y = -ab\left(\frac{X^*}{Y^* + bX^*}\right)^2$,

$z = c\left(\frac{Y^*}{Y^* + bX^*}\right)^2$,

$w = s\left(1 - \frac{2Y^*}{K_2}\right) - bc\left(\frac{X^*}{Y^* + bX^*}\right)^2$.

It is clear that $y < 0$ and $z > 0$. By substituting (6) into w, it is readily seen that $w < 0$. Under condition (2) and (4), by substituting (5) into x, we get $x < -r\left(\frac{N^*}{K_1}\right)^2 < 0$ whereas under condition (3), x can be negative or positive. For condition (2) and (4), theorem 4.3 is fulfilled because x, y and w are negatives and $z > 0$. Hence, E_3 (X^*, Y^*) islocally asymptotically stable if condition (2) or (4) satisfied.

CONCLUSION

The model of predator-prey ratio-dependent response function is a system of two-dimensional nonlinear ordinary differentialequations. The system has three equilibrium point, namely the prey extinction point E_1 (0, K_2), the predator extinction pointE_2 (K_1, 0) and the survival pointE_3 (X^*, Y^*). Based on the analysis, E_2 (K_1, 0) is unstable. While, E_1 (0, K_2) and E_3 (X^*, Y^*) are local asymptotically stablewith certain conditions.

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